Influence of Wavelength Interchange on the Blocking Performance of Wavelength Routed Chordal Ring Networks

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Abstract

In this paper, we investigate the influence of wavelength interchange on the blocking performance of wavelength routed optical networks with chordal ring topology. We have considered chordal rings with chord lengths of 3, N/4, and \(\sqrt{N} + 3\), being \(N\) the number of nodes. It is shown that chordal ring networks with a chord length of \(\sqrt{N} + 3\) and mesh-torus networks have a similar behaviour regarding wavelength interchange.

I. INTRODUCTION

Internet traffic is increasing drastically, as shown in Figure 1, due to the rise of new communication services and due to the explosive growth of the number of hosts [1]-[2]. To satisfy the bandwidth requirements that enable those new services and a higher number of users, WDM systems are being deployed world wide, which is seen as the first phase of optical networking. After this, the introduction of optical add/drop multiplexers in a linear architecture and the use of WDM protection switches are expected. This architecture will rapidly evolve to the WDM ring architecture, and a further possible evolution scenario may be the interconnection of WDM rings and mesh networks [3]. Whereas the evolution from the point-to-point WDM transmission system to interconnected rings is clear from a physical topology point of view, the optimal topology to be used for the mesh network is a less studied subject. In [4], a study is presented of the influence of node degree on the fibre length, capacity utilisation, and average and maximum path lengths of wavelength routed mesh networks. It is shown that average node degrees varying between 3 and 4.5 are of particular interest.

In this paper we consider chordal rings, which are a particular family of regular graphs of degree 3 [5]. In [6], we have analysed the influence of chord length on the blocking performance of wavelength routed chordal ring networks and we have observed that chordal rings with a minimum network diameter have a blocking performance close to the performance of mesh-torus, which, in turn, have a node degree of 4. Thus, the choice of chordal ring networks with minimum network diameter, instead of mesh-torus networks, leads to a reduction in the number of links, and hence in the total cable length, since the number of links in a \(N\)-node chordal ring is \(3N\), and the number of links in a \(N\)-node mesh-torus is \(4N\).

In this paper, we investigate the influence of wavelength interchange on the blocking performance of chordal ring networks.

Fig. 1. Forecast of traffic growth. Data provided in [2].
1 Petabyte=10^{15} bytes.

The remainder of this paper is organised as follows. The analytical model used to compute the blocking probability is briefly described in section II. The assessment of the blocking performance in wavelength routed chordal ring networks is presented in section III. Main conclusions are presented in section IV.

II. ANALYTICAL MODEL USED TO EVALUATE THE PATH BLOCKING PROBABILITY

A chordal ring is basically a ring network in which each node has an additional link, called a chord. The number of nodes in a chordal ring is assumed to be even and nodes are indexed 0, 1, 2, …, \(N-1\) around the \(N\)-node ring. It is also assumed that each odd-numbered node \(i (i=1, 3, \ldots, N-1)\) is connected to a node \((i+w) \mod N\), where \(w\) is the chord length, which is assumed to be positive odd and, without loss of generality, we also assume that \(w \leq N/2\), as in [5].

For a given number of nodes \(N\), several chordal rings can be obtained for different values of the chord length. Figure 2
shows a chordal ring with 20 nodes and a chord length of 3.

According to Theorem 2 presented in [5], the diameter of a chordal ring network with \( N \) nodes, where \( N \) is a square and \( N \geq 8 \), is greater than or equal to \( \sqrt{N} + 3 \). This equality holds when \( \sqrt{N} + 3 \leq w \leq \sqrt{N} + 2h + 1 \), where \( h \) is the largest integer that satisfies \( \sqrt{N} \geq 2(2h^2 + h + 1) \). This Theorem is particularly important since it allows the identification of chord lengths that lead to the smallest network diameter.

Fig. 2. Chordal ring network with 20 nodes and a chord length of 3.

To compute the blocking probability in wavelength routed chordal ring networks, we have used the model given in [7], since it applies to ring topologies, has a moderate computational complexity, and takes into account dynamic traffic and the correlation between the wavelengths used on successive links of a multi-link path.

The following assumptions are used in the model [7]: 1) Call requests arrive at each node according to a Poisson process with rate \( \lambda \), with each call equally likely to be destined to any of the remaining nodes; 2) Call holding time is exponentially distributed with mean \( 1/\mu \), and the offered load per node is \( \rho = \lambda/\mu \); 3) The path used by a call is chosen according to a pre-specified criterion (e.g. random selection of a shortest path), and does not depend on the state of the links that make up a path; the call is blocked if the chosen path cannot accommodate it; alternate path routing is not allowed; 4) The number of wavelengths per link, \( F \), is the same on all links; each node is capable of transmitting and receiving on any of the \( F \) wavelengths; each call requires a full wavelength on each link it traverses; 5) Wavelengths are assigned to a session randomly from the set of free wavelengths on the associated path.

Besides these assumptions, it is also assumed that given the loads in links 1, 2, ..., \( i-1 \) of a path with \( i \) links, the load in link \( i \) depends only on the load in link \( i-1 \). This model also assumes that that the hop-length distribution is known, as well as the arrival rates of calls at a link that continue, and those that do not, to the next link of a path. The call arrival rates at links have been estimated from the arrival rates of calls to nodes, as in [7]. The hop-length distribution is a function of the network topology and the routing algorithm. Using the shortest-path algorithm, in a \( N \)-node chordal ring with chord length \( w \), it was not possible to obtain a general expression for the hop-length distribution. However, we have found analytical expressions for the hop-length distribution when the chord length is 3 (\( w=3 \)), when the chord length is maximum (\( w=N/2 \) or \( w=N/2-1 \)), when the chord length is as close as possible to the mean chord length (\( w=N/4 \)), and for a chord length that leads to the smallest network diameter (\( w=\sqrt{N} + 3 \)). Hop-length distributions for these cases are presented below.

If \( N=6+4k \) (\( k=0, 1, 2, \ldots \)) and for \( w=3 \) or \( w=N/2 \),

\[
p_l = \begin{cases} 
\frac{3}{N-1}, & \text{for } l=1 \\
\frac{4}{N-1}, & \text{for } 2 \leq l \leq \left(\frac{N+2}{4}-1\right) \text{ and } N \geq 10 \\
\frac{2}{N-1}, & \text{for } l = \frac{N+2}{4} 
\end{cases}
\] (1)

If \( N=8+4k \) (\( k=0, 1, 2, \ldots \)) and for \( w=3 \),

\[
p_l = \begin{cases} 
\frac{3}{N-1}, & \text{for } l=1 \\
\frac{4}{N-1}, & \text{for } 2 \leq l \leq \left(\frac{N+1}{4}-1\right) \text{ and } N \geq 12 \\
\frac{3}{N-1}, & \text{for } l = \frac{N}{4} \\
\frac{1}{N-1}, & \text{for } l = \frac{N}{4} + 1 
\end{cases}
\] (2)

If \( N=16+4k \) (\( k=0, 1, 2, \ldots \)) and for \( w=N/2-1 \),

\[
p_l = \begin{cases} 
\frac{3}{N-1}, & \text{for } l=1 \\
\frac{5}{N-1}, & \text{for } l = 2, 3 \\
\frac{4}{N-1}, & \text{for } 4 \leq l \leq \left(\frac{N}{4}-1\right) \text{ and } N \geq 20 \\
\frac{2}{N-1}, & \text{for } l = \frac{N}{4} 
\end{cases}
\] (3)
If \( N = 64+8k \) \((k=0, 1, 2, \ldots)\) and for \( w = N/4 +1 \),

\[
 p_l = \begin{cases} 
 \frac{3l}{N-1}, & \text{for } 1 \leq l \leq 3 \\
 \frac{11}{N-1}, & \text{for } l = 4, 5 \\
 \frac{16-l}{N-1}, & \text{for } l = 6, 7 \\
 \frac{8}{N-1}, & \text{for } 8 \leq l \leq \left(\frac{N-1}{8}\right) \text{ and } N \geq 72 \\
 \frac{4}{N-1}, & \text{for } l = \frac{N}{8} 
\end{cases} 
\]

(4)

If \( N = 36+8k \) \((k=0, 1, 2, \ldots)\) and for \( w = N/4 \),

\[
 p_l = \begin{cases} 
 \frac{3l}{N-1}, & \text{for } 1 \leq l \leq 3 \\
 \frac{9}{N-1}, & \text{for } l = 4 \\
 \frac{8}{N-1}, & \text{for } 5 \leq l \leq \frac{N-4}{8} \text{ and } N \geq 44 \\
 \frac{6}{N-1}, & \text{for } l = \frac{N+4}{8} \\
 \frac{2}{N-1}, & \text{for } l = \frac{N+12}{8} 
\end{cases} 
\]

(5)

If \( N = 42+8k \) \((k=0, 1, 2, \ldots)\) and for \( w = (N/2 +1)/2 \),

\[
 p_l = \begin{cases} 
 \frac{3l}{N-1}, & \text{for } 1 \leq l \leq 3 \\
 \frac{14-l}{N-1}, & \text{for } l = 4, 5 \\
 \frac{8}{N-1}, & \text{for } 6 \leq l \leq \frac{N-2}{8} \text{ and } N \geq 50 \\
 \frac{4}{N-1}, & \text{for } l = \frac{N+6}{8} 
\end{cases} 
\]

(6)

If \( N = 46+8k \) \((k=0, 1, 2, \ldots)\) and for \( w = (N/2 -1)/2 \),

\[
 p_l = \begin{cases} 
 \frac{3l}{N-1}, & \text{for } 1 \leq l \leq 3 \\
 \frac{14-l}{N-1}, & \text{for } l = 4, 5 \\
 \frac{8}{N-1}, & \text{for } 6 \leq l \leq \frac{N-6}{8} \text{ and } N \geq 54 \\
 \frac{6}{N-1}, & \text{for } l = \frac{N+2}{8} \\
 \frac{2}{N-1}, & \text{for } l = \frac{N+10}{8} 
\end{cases} 
\]

(7)

For \( N = m^2 \), with \( m=10+2k \) \((k=0, 1, 2, 3, \ldots)\), and for \( w = \sqrt{N} +3 \),

\[
 p_l = \begin{cases} 
 \frac{3l}{N-1}, & \text{for } 1 \leq l \leq \frac{m+1}{2} \\
 \frac{2m+6-l}{N-1}, & \text{for } \frac{m+2}{2} \leq l \leq m-4 \text{ and } N \geq 144 \\
 \frac{m+14}{2N-1}, & \text{for } l = m-3 \\
 \frac{13}{N-1}, & \text{for } l = m-2 \\
 \frac{4}{N-1}, & \text{for } l = m-1
\end{cases} 
\]

(8)

Besides the blocking probability, for analysis of the benefits of wavelength conversion, two metrics are usually used to quantify the wavelength conversion gain: the blocking probability gain, which corresponds to a reduction in blocking probability, and the utilisation gain, which measures the increase in network utilisation [8]. Here, we concentrate only on blocking probability gain.

III. ASSESSMENT OF BLOCKING PERFORMANCE

In this section, we investigate the influence of wavelength interchange on the blocking performance of chordal ring networks. The performance assessment is focused on chordal rings with chord lengths of 3, \( N/4 \), and \( w = \sqrt{N} +3 \), being \( N \) the number of nodes in the network. The last chord length is very important since it leads to the smallest network diameter. If the number of nodes is larger than or equal to 484, there are other chord lengths that also lead to the smallest network diameter, but here we concentrate only on the above referred chord lengths.

Figure 3 shows the blocking probability for 100-node chordal ring networks with chord lengths of 3, \( N/4 \), or \( \sqrt{N} +3 \), and with a load per node of 0.5 Erlang. As can be seen, as the converter density increases from 0 to 1, the slope of the curves is nearly independent of the chord length, but the use of the chord length of \( \sqrt{N} +3 \) leads to the largest reduction in blocking probability, as the number of wavelengths increases from 8 to 16.

Figure 4 shows the blocking probability gain due to wavelength interchange, as a function of the load per node, for chordal ring networks with 100 nodes. As can be seen, when the load per node is low, the highest gains are obtained with a chord length of 3 and the lowest gains are obtained with a chord length of \( w = \sqrt{N} +3 \). However, the inverse situation is observed for higher loads per node: the highest gains are obtained with a chord length of \( w = \sqrt{N} +3 \) and the
lowest gains are obtained with a chord length of 3 (see Figure 4). From Figure 5, we may also see that, for a load of 0.5 Erlang, the highest gain is obtained for a chord length of 3 when the number of wavelengths per link is larger than or equal to 15.

Figure 6 shows the blocking probability, as a function of the converter density, for chordal ring networks with \( w = \sqrt{N} + 3 \) and mesh-torus networks, both with 100 nodes and a load per node of 0.5 Erlang. As may be seen, for each number of wavelengths per link, the influence of wavelength

Fig. 3. Blocking probability for chordal rings with 100 nodes and a load per node of 0.5 Erlang. 
\( w \): chord length; \( F \): number of wavelengths per link.

\[ \cdots w=3, F=8 \]
\[ w=3, F=16 \]
\[ w=N/4, F=8 \]
\[ w=N/4, F=16 \]
\[ w=\sqrt{N}+3, F=8 \]
\[ w=\sqrt{N}+3, F=16 \]

Fig. 4. Blocking gain due to wavelength interchange for 100-node chordal ring networks. 
\( w \): chord length; \( F \): number of wavelengths per link.

\[ \cdots w=3, F=8 \]
\[ w=3, F=16 \]
\[ w=N/4, F=8 \]
\[ w=N/4, F=16 \]
\[ w=\sqrt{N}+3, F=8 \]
\[ w=\sqrt{N}+3, F=16 \]

Fig. 5. Blocking gain due to wavelength interchange, as a function of the number of wavelengths per link, for 100-node chordal ring networks with a load per node of 0.5 Erlang.

\[ \cdots \]

interchange on both networks is very similar: curves have a similar shape and are relatively close. We have increased further the number of nodes, to 1600. Figure 7 shows the influence of converter density on the blocking probability for mesh-torus and chordal rings with \( w = \sqrt{N} + 3 \), both with 1600 nodes and a load per node of 0.1 Erlang. Again, the

Fig. 6. Blocking probability, as a function of the converter density, for chordal ring networks with \( w = \sqrt{N} + 3 \) and mesh-torus networks, both with 100 nodes and a load per node of 0.5 Erlang.

\[ \cdots \]

Chordal ring, \( F=8 \)
Chordal ring, \( F=12 \)
Chordal ring, \( F=16 \)
Mesh-torus, \( F=8 \)
Mesh-torus, \( F=12 \)
Mesh-torus, \( F=16 \)
influence of wavelength interchange on both networks is very similar. Moreover, in this case wavelength interchange is very helpful as the converter density increases from 0 to 1. This means that wavelength conversion leads to larger performance improvements on chordal rings, with $w = \sqrt{N} + 3$, or mesh-torus networks, both with a large number of nodes.

It is well known [7] that, in rings, only a small fraction of nodes needs to be equipped with wavelength converters, in order to obtain a blocking performance similar to the one obtained when all nodes are equipped with wavelength converters. As in rings, the use of wavelength interchange improves the blocking performance in chordal rings, with $w = \sqrt{N} + 3$, as the number of wavelengths increases, but unlike rings, the difference between blocking probabilities, with and without wavelength interchange, increases as the number of nodes increases. Moreover, the performance of chordal rings with $w = \sqrt{N} + 3$ is similar to the performance of mesh-torus networks.

![Fig. 7. Blocking probability versus converter density for chordal ring networks with $w = \sqrt{N} + 3$ and mesh-torus networks, both with 1600 nodes and a load per node of 0.1 Erlang.](image)

IV. CONCLUSIONS

Using an analytical model, we have investigated the influence of wavelength interchange on the blocking performance of wavelength routed chordal ring networks with chord lengths of 3, $N/4$, and $\sqrt{N} + 3$. We have found that, in 100-node chordal ring networks with chord lengths of 3, $N/4$, or $\sqrt{N} + 3$, and with a load per node of 0.5 Erlang, the slope of the curves is nearly independent of the chord length, but the use of the chord length of $\sqrt{N} + 3$ leads to the largest reduction in blocking probability. We have also found that the performance of chordal rings with a chord length of $\sqrt{N} + 3$ is similar to the performance of mesh-torus networks. Besides, unlike ring networks, in chordal rings with a chord length of $\sqrt{N} + 3$ and mesh-torus networks, both with a large size, wavelength interchange is very helpful as the converter density increases from 0 to 1, leading to large performance improvements.

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