SAR moving targets processing for low resources platforms

Paulo Marques(a),(b) and José Dias(b)

(a) Instituto Superior de Engenharia de Lisboa, ISEL-DEETC, Gal. 16, R. Conselheiro Emídio Navarro 1, 1959-007 Lisboa, Portugal
Fax: +351.21.8317114, E-mail: pmarques@isel.ipl.pt

(b) Instituto Superior Técnico - Instituto de Telecomunicações, IST-IT, Torre Norte, Piso 10, Av. Rovisco Pais, 1049-001 Lisboa, Portugal
Fax: +351.21.8418472, E-mail: bioucas@lx.it.pt

Abstract - This paper presents a methodology to retrieve the full velocity vector of moving targets inducing Doppler-shifts beyond the Nyquist limit determined by the pulse repetition frequency (PRF). The linear dependency of the Doppler-shift with respect to the slant-range velocity, at each wavelength, is exploited in the proposed scheme. The full velocity vector is unambiguously estimated using data from a single synthetic aperture radar (SAR) sensor, thus solving the so-called blind angle ambiguity. The proposed approach is very efficient from the computational and hardware point of view, being therefore adequate for use in low resources platforms.

I. Introduction

Unmanned Air Vehicle (UAV) systems demand low weight payloads. UAVs are used in the battlefield with SAR sensors that can be used to detect moving targets and estimate their velocity. The problem of estimating the full trajectory parameters of moving targets using SAR data typically requires two or more channels [1]. Herein, we present a novel technique to estimate the full velocity vector of moving targets, that is light from the computational point of view and has low hardware needs since it uses data from a single SAR channel. The requirement of a single SAR channel contributes to the decrease of the UAV payload, or, alternatively, enables the incorporation of other sensors thus enhancing the utility of the overall system.

A moving target induces, in the SAR returned signal, a Doppler-shift and a Doppler-spread in the slow-time frequency domain. Assuming a broadside geometry, the cross-range and slant-range velocities of a moving target are responsible for the spread and for the Doppler-shift, respectively, both in the slow-time frequency domain. Given a pulse repetition frequency (PRF), the Doppler-shift $f_D = 2v_x/\lambda$, where $v_x$ is the target slant-range velocity and $\lambda$ is the signal wavelength, is confined to $|f_D| < PRF/2$. If the induced Doppler-shift exceeds $PRF/2$ it has been mostly accepted that the true moving target slant-range velocity cannot be uniquely determined using a single antenna and a single pulse scheduling [2], [3]. Classical solutions to process such targets with a single antenna consist in increasing the PRF [2] or, alternatively, in using a non-uniform PRF as proposed in [3] and [4]. Increasing the PRF shortens the unambiguous range swath and increases the memory requirements to store the received signal. The use of a non-uniform PRF requires a non-conventional pulse scheduling, introducing complexity in image reconstruction algorithms.

The present paper elaborates on ideas presented in [1] and [5] and proposes a novel technique to estimate the full velocity vector of ground-based moving targets with velocities above the Nyquist limit. It uses the SAR ambiguity function proposed by Soumekh in [6, Ch. 6] to detect and estimate velocity magnitude of the moving object. Then, by taking advantage of the linear dependence of the Doppler-shift on the slant-range velocity, at each fast-time frequency, the unaliased estimate of the slant-range velocity is obtained [1]. The cross-range velocity is computed from the previous two estimates. For scenes with signal-to-clutter ratio (SCR) higher than, roughly, 10dB, the proposed scheme is effective, even when the ground echoes are completely superimposed, in the frequency domain, on the moving objects echoes.

This paper is organized as follows. In Section 2 we review basic properties of moving target echoes in SAR and present the proposed methodology to retrieve unaliased estimates of the slant-range velocity. In Section 3 we show results taking real and simulated data to illustrate the effectiveness of the proposed scheme.

II. Proposed Approach

Let us consider the SAR scenario illustrated in Fig. 1, where the radar travels at constant velocity $V$ and constant altitude along the flight path (cross-range direction). The antenna transmits microwave pulses and records the backscattered echoes. The illuminated scene contains a moving target with reflectivity $f_m$, slant-plane coordinates $(x_0, y_0)$ when the SAR platform is at position $u = u_0$, and velocity vector $(-v_x, -v_y)$.

---

1This work was supported by the Fundação para a Ciência e Tecnologia, under the project POSI/34071/CPS/2000 and by Instituto Politécnico de Lisboa under the project 5826/2004.
Let \( p(t) \) be the transmitted pulse when the antenna is at cross-range coordinate \( u \) and \( S_m(u, t) \) the corresponding received signal. The 2D Fourier transform of the received signal, \( S_m(k_u, k) \), where \( k_u = 2\pi/\lambda_u \) is the slow-time frequency domain and \( k = 2\pi/\lambda \) is the fast-time frequency domain is, after pulse compression, given by [7], [8]

\[
S_m(k_u, k) = |P(\omega)|^2 A(k_u, k) f_m \times e^{-j\sqrt{4k^2 - (\frac{2\pi}{\lambda_u})^2}} x e^{-j\frac{2\pi}{\lambda_u}Y},
\]

where \( P(\omega) \) is the Fourier transform of the transmitted pulse \( p(t) \) and \( \omega = 2\pi/\lambda \). Symbols \( \lambda \) and \( \lambda_u \) denote the signal wavelength in the fast-time and in the slow-time frequency domain, respectively, and \( c \) is the speed of light. Function \( A(k_u, k) \) is the two-way antenna radiation pattern. Symbol \( \alpha = \sqrt{\rho^2 + \nu^2} \) is the relative speed of the moving target with respect to the radar, where \( \nu \equiv (1 + \nu_y/V) \) and \( \mu = \nu_y/V \) denote, respectively, the moving target relative cross-range and slant-range velocities, with respect to the sensor velocity.

In [8], we have shown that the amplitude modulation term \( A(k_u, k) \) of the returned echo from a moving target, in the two-dimensional frequency domain, the shape of the two-way antenna radiation pattern according to

\[
A(k_u, k) \propto g^2 \left( \frac{1}{2\rho^2} (k_u - 2k\mu) \right),
\]

valid for \( \nu_y > -V \), where \( g \) is related with the 2D Fourier transform of the electric field at the antenna aperture. Relatively to a static target, and for a constant wavenumber \( k \), the shape \( g \) becomes dilated by \( 2\nu \) and shifted by \( 2\mu \). If the transmitted pulse has bandwidth \( B \), then \( k \) is confined to

\[
k_{\text{min}} = \frac{\pi B}{c} + k_0 < k \leq \frac{\pi B}{c} \equiv k_{\text{max}},
\]

where \( k_0 = 2\pi/\lambda_0 \) and \( \lambda_0 \) is the carrier wavelength. For a moving target with relative slant-range velocity \( \mu \), we see from expression (2) that the support of the returned signal \( S_m(k_u, k) \) exhibits a slope of \( 2\mu \) with respect to the \( k \) axis, as illustrated in Fig. 2. In this figure, \( k_{u_{\text{end}}} \) and \( k_{u_{\text{start}}} \) denote the Doppler-shifts at the fast-time frequencies \( k_{\text{max}} \) and \( k_{\text{min}} \), respectively. We conclude then that

\[
\mu = \frac{k_{u_{\text{end}}} - k_{u_{\text{start}}}}{2(k_{\text{max}} - k_{\text{min}})},
\]

regardless of the PRF.

In the absence of electronic noise and ground clutter, \( k_{u_{\text{end}}} \) and \( k_{u_{\text{start}}} \) could be inferred using a simple centroid technique. This solution cannot, however, be applied in the case of ground moving targets, because the returned signal coexists with clutter returns in the 2D spectrum. The weight of the clutter can be reduced by spotlighting the moving target area [6, Ch. 6]. Once the moving target signature is spotlighted in the spatial domain, it is resynthesized back to the 2D frequency domain for further processing.

We have shown in [1] that, under certain circumstances, the correlation of the static ground returns, in the \( (k_u, k) \) domain, decays very quickly in both dimensions. Concerning moving targets, the same is not true. By exploiting these distinct statistical properties between the signals echoed by the moving targets and those of echoed by the clutter we will derive a methodology to unambiguously estimate the moving targets slant-range velocities and also retrieve their full velocity vector.

### A. Moving target signature properties

The autocorrelation function \( R_{SS}(\Delta k_u, k_1, k_2) \) between \( S_m(k_u, k_1) \) and \( S_m(k_u, k_2) \) with respect to \( k_u \) is [1]

\[
R_{SS}(\Delta k_u, k_1, k_1 + \Delta k) \approx |P(\omega_1)|^2 |P(\omega_2)|^2 |f_m| e^{-j\left[\frac{\Delta\omega}{\Delta k} Y - \frac{\Delta\omega^2}{4(k_1 + \Delta k)^2} X\right]} \times \int_{-\infty}^{+\infty} A(k_u, k) A^*(k_u - 2\Delta k\mu - \Delta k_u, k_1) e^{j\phi} dk_u, \tag{5}
\]

where \( \omega_1 = k_1 c \), \( \omega_2 = k_2 c \), \( \Delta k = k_2 - k_1 \), and

\[
\phi = \frac{2k_1 \Delta k_u}{4(k_1 + \Delta k)^2} X. \tag{6}
\]

If phase \( \phi \) has an excursion smaller than \( \pi \) in the Doppler interval equivalent to the antenna bandwidth, the last line of (5)
is a correlation between \( A(k_u, k_1) \) and \( A(k_u - 2\Delta k, k_1) \), with respect to \( k_u \), computed at \( \Delta k_u \). The correlation magnitude \( |R_{SS}(\Delta k_u, k_1, k_1 + \Delta k)| \) exhibits a maximum that is linearly dependent on \( \Delta k \) by a factor of 2\( \mu \).

The maximum relative slant-range velocity that can be estimated using this methodology is thus imposed by the above referred restriction on phase \( \phi \) and is given by

\[
|\mu| < \frac{(k + \Delta k)\pi}{B_A \Delta k X},
\]

where \( \Delta k_u = 2\Delta k \mu \) and \( B_A \) is the two-way antenna bandwidth. Discussion on how to make this bound larger can be found in [1].

The cross-range velocity component is obtained using the value \( \alpha \) given by the SAR ambiguity function [6, Ch. 6] and the estimated \( \mu \) given by the spectral skew,

\[
\nu = \sqrt{\alpha^2 - \mu^2}.
\]  

\[ (8) \]

B. Proposed methodology

The strategy proposed to estimate the full velocity vector of the moving objects is now summarized.

1) Compute the SAR ambiguity function to detect the moving targets and estimate the corresponding \( \alpha \) and coordinates \((X, Y)\).

2) For each detected moving target:

2.1) Digitally spotlight the moving target image in the spatial domain and re-synthesize its signature back to the \((k_u, k)_1\), obtaining the signal \( S_m(k_u, k) = S_m(k_u, k) + S_{0R}(k_u, k) \), where \( S_{0R} \) denotes the remaining noise after the digital spotlight operation.

2.2) Compensate phase \( \phi \) using the target approximate slant-range coordinate \( X \) estimated in step 1) and the estimated \( \alpha \).

This is accomplished by multiplying \( \hat{S}_m(k_u, k) \) by

\[ \exp\{j\sqrt{4k^2 - (k_u/\alpha)^2}X\}. \]

2.3) Compute the correlation \( R_{SS} \) between \( \hat{S}_m(k_u, k_0) \) and \( \hat{S}_m(k_u, k) \) for a set of discrete wavenumbers within the transmitted pulse bandwidth. As shown in (5), \( |R_{SS}| \) will display a maximum for each \( k \) at \( \Delta k_u = 2(k - k_0)\mu \).

2.4) Perform a linear regression on the ordinates corresponding to the maximum values of \( |R_{SS}| \) to estimate \( \mu \) and subsequently compute the target slant-range velocity.

2.5) Estimate \( \nu = \sqrt{\alpha^2 - \mu^2} \) and compute the target cross-range velocity.

To evaluate the computational complexity of the proposed methodology, we follow a strategy similar to that presented in [9, Ch. 12]. The approach consists in estimating the number of complex operations \( (C_{ops}) \) for each major step of the algorithm. A complex operation is defined as one radix-2 FFT butterfly, which consists of ten floating point operations (four floating point multiplications and six floating point additions). An equal cost for multiplications and additions is assumed. Accordingly to [9, Ch. 12.2] and [10, Ch. 15], the following \( C_{ops} \) are accounted:

- FFT of size \( N \): \( C_{fft} \approx N/2 \log_2 N \) \([C_{ops}]\);
- 2D FFT with dimension of \( N_x \) by \( N_y \): \( C_{fft} \approx N_xN_y/5 \log_2 N_xN_y \) \([C_{ops}]\);
- Complex multiplication: \( C_m \approx 1 \) \([C_{ops}]\);
- Complex-by-real multiplication: \( C_{mr} \approx 0.5 \) \([C_{ops}]\);
- 2D linear interpolation with size of \( N_x \) by \( N_y \): \( C_p \approx 2N_xN_y \) \([C_{ops}]\).

The number of \( C_{ops} \) just accounted, slightly overestimates the total number of operations for a given algorithm. In this way we provide a margin for unaccounted machine cycles used in operations such as array index generation and memory access.

By summing the number of \( C_{ops} \) for the proposed strategy, we obtain,

\[
C \approx 2N(2/5 \log_2 N + 1) + N_{targets} \times \left[ N_s \left( \frac{6}{5} \log_2 N_s + \frac{5}{2} \right) + \frac{\sqrt{N_s}}{10}(7 + \sqrt{N_s}) \right] [C_{ops}],
\]

where \( N \equiv N_x \times N_y \) and \( N_s \equiv N_{xs} \times N_{ys} \); symbol \( N_{targets} \) denotes the number of moving targets to process. For step 2.1 of the algorithm, we assumed \( N_{xs} \) correlations, each one of size \( N_{ys} \). To obtain a simpler expression, we also considered \( N_x = N_y \) and \( N_{xs} = N_{ys} \).

As a numerical example, let us consider a target area of size \( N = 1024 \times 1024 \) pixels, containing \( N_{targets} = 100 \) targets and that each digitally spotlighted region is of size \( N_s = 20 \times 20 \) pixels. In this situation, the algorithm requires 19.3 millions of \( C_{ops} \), which is accomplished in less than half a second by a conventional desktop computer with a processor running at a clock speed of 1.5GHz.

III. Estimation Results

The proposed scheme is now applied to MSTAR data. The experiments include clutter from Huntsville-Alabama and three BTR-60 transport vehicles with simulated movement. All the BTR-60 move with velocities several times above the Nyquist limit. The mission parameters are presented in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency</td>
<td>9.6GHz</td>
</tr>
<tr>
<td>Chirp bandwidth</td>
<td>250MHz</td>
</tr>
<tr>
<td>Altitude</td>
<td>12km</td>
</tr>
<tr>
<td>Velocity</td>
<td>637km/h</td>
</tr>
<tr>
<td>Look angle</td>
<td>15°</td>
</tr>
<tr>
<td>Antenna radiation pattern</td>
<td>Raised Cosine</td>
</tr>
<tr>
<td>Oversampling factor (cross-range)</td>
<td>2</td>
</tr>
<tr>
<td>PRF</td>
<td>177Hz</td>
</tr>
</tbody>
</table>

Table 1: Mission parameters.
Table 2 details the moving targets velocities \((v_x, v_y)\) and initial coordinates \((x_0, y_0)\). The SCR is roughly set to 23 dB. Notice that the slant-range velocities of both targets induce Doppler-shifts ranging from 6.25 to 12 times the maximum unambiguous value imposed by the PRF.

<table>
<thead>
<tr>
<th>Target</th>
<th>(x_0) [m]</th>
<th>(y_0) [m]</th>
<th>(v_x) [m/s]</th>
<th>(v_y) [m/s]</th>
<th>(v_{max})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>75</td>
<td>220</td>
<td>8.64</td>
<td>-10</td>
<td>6.25</td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>122</td>
<td>16.58</td>
<td>-2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>220</td>
<td>115</td>
<td>11.74</td>
<td>-2</td>
<td>8.5</td>
</tr>
</tbody>
</table>

Table 2: BTR-60 Transport vehicle trajectory parameters.

The resulting data was focused using the wavefront reconstruction algorithm with static ground parameters. The obtained image is presented in Fig. 3, where the moving objects appear defocused and misplaced as expected.

The methodology presented in [8], at expenses of higher computational requirements. Besides that, and in contrast with the methodology herein proposed, that strategy requires a good estimate of the antenna pattern.

### IV. Conclusions and Final Remarks

This paper presents a novel methodology to retrieve the full velocity vector of moving targets inducing Doppler-shifts beyond the Nyquist limit imposed by PRF. The methodology exploits the linear dependency of the Doppler-shift with the slant-range velocity for each fast-time frequency. By combining the developed scheme with an existing algorithm to detect moving targets and estimate their velocity vector magnitude, the full velocity vector is estimated with good accuracy using aliased data from a single SAR sensor. The proposed methodology is very efficient from the computational point of view and uses data from a single channel SAR sensor, being therefore adequate for use in low resource platforms such as UAVs.

### References


