Performance Analysis of OFDM Polarization Receive Diversity System in Correlated Ricean Fading Channels

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Abstract – In this paper a complete analytical model for BER (Bit Error Rate) calculation of OFDM (Orthogonal Frequency Division Multiplexing) system with polarization diversity is presented. The assumed system consists of one transmit and dual polarized receive antenna. Propagation environment is characterized with Ricean fading and novel method for analytical determination of the system BER performance is introduced. Maximal ratio combining of the received diversity signals is performed on subcarrier basis. The obtained analytical results are compared with the ones attained by simulation. It is shown that polarization diversity scheme can be considered as an efficient solution for improving OFDM system performance.

I. INTRODUCTION

The rapid growth of telecommunication industry is related with increasing demand for a variety of multimedia services. That is why a significant focus is on future broadband wireless networks that will provide high data rates and sufficient system capacity. Performance of these systems can be limited by random fluctuations in amplitude of the received signal caused by multipath propagation, called fading. To overcome such problem, various techniques for performance improvement are proposed. In this paper we focus on the combination of two such powerful techniques, OFDM (Orthogonal Frequency Division Multiplexing) and diversity.

OFDM transmission scheme enables efficient use of the limited frequency spectrum, providing high data rates by splitting the data stream into N parallel streams with reduced data rates. Using orthogonal subcarriers, spectral overlapping is allowed, therefore obtaining better spectral efficiency. Furthermore, using properly chosen guard interval and cyclic prefix, intersymbol interference (ISI) caused by multipath reception can be minimized.

Despite the above mentioned advantages of OFDM systems, it has been shown that further enhancements regarding the received signal quality are necessary [1]. Antenna diversity is an effective way to achieve such a goal.

The most commonly used diversity technique in the present wireless communication systems is space diversity. There, for reaching a full diversity gain, when receive diversity antennas must be adequately separated, i.e. at least half wavelength, when implemented at the mobile unit, and tens wavelengths at the base station. At the same time, a space for mounting two or more antennas can be limited, especially in urban environments. A promising alternative to such a solution is polarization diversity. Since it utilizes one dual polarized antenna, this type of diversity is space and cost effective solution despite the fact that the antenna collocation necessary leads to signal correlation. However, studies have shown that multiple antenna based systems can achieve a significant diversity gain as long as the correlation coefficient is less than 0.7 [2]. When polarization diversity is considered, this requirement is almost always fulfilled. In fact, experimental results have shown that envelope correlation coefficient is generally even less than 0.2 [3].

Polarization diversity is based on the fact that propagation characteristics in wireless communication system are different for vertically and horizontally polarized waves. Multiple reflections between the transmitter and the receiver lead to the depolarization of radio waves, coupling some energy of the transmitted signal into the orthogonal polarized wave. Due to that characteristic of multipath radio channel, vertically/ horizontal polarized transmitted waves have also horizontal/ vertical component. In the case of insufficient depolarization, significant power imbalance between the average power of the copolarized and crosspolarized signals can make the whole diversity system worthless. Parameter that indicates this power difference is denoted as cross-polar discrimination (XPD). Its influence on system performance is greater than that of the correlation of received signals. Generally, it can be shown that high XPD values lead to significant degradation of the system performance, i.e. lower diversity gain. Typical values of this parameter vary from 1-10 dB in urban and suburban environment, and 10-18dB in rural environment [3].

In this paper we analyze the case when OFDM system is implemented in combination with the polarization diversity, which can be considered as an optimal diversity technique taking into account the performance improvements as well as space and material resources required.

The nature of the electromagnetic wave is such that a signal which arrives orthogonal to the obstacle surface is attenuated more than the signal parallel to the surface. Since buildings are typical obstacles in wireless channel, it is expected that horizontally polarized wave (Hpol) is more attenuated than vertically polarized wave (Vpol). That is why common system architecture assumes vertically polarized transmit antenna [3] and dual polarized receive antenna. Assuming that the signal at the receiver consists of one dominant and several reflected signal components, envelope fluctuation are described with Ricean PDF (Probability Density Function). Taking into account the existence of the signal correlation between branches, we propose an analytical method for transforming the correlated signals into uncorrelated ones, in order to analytically determine BER performance of the analyzed system. In that manner, effects of correlation are expressed through modification of the following system parameters: the average received signal–to–noise ratio (SNR) per diversity branch and Ricean K factor. Further analysis implies standard BER (Bit Error Rate) calculation procedure applied to uncorrelated signals. It is assumed that input data streams are mapped according to BPSK (Binary Phase Shift
Keying), and maximal ratio combining (MRC) of the received diversity signals is applied. The analytically obtained results are then compared with the ones produced by a simulation. It is clearly shown that, under the described propagation conditions, polarization diversity in combination with OFDM can be considered as an effective solution for the performance improvement of the broadband wireless systems.

This paper is organized as follows. After introduction, in Section 2 we describe the system model, and propose transformation that enables decorrelation of two mutually correlated signal envelopes. A complete analytical model for BER calculation of the OFDM polarization diversity system is also derived. Section 3 presents results obtained by the proposed analytical model, as well as their comparison with simulation results. Finally, conclusion is given in Section 4.

II. SYSTEM MODEL

In order to analytically determine error performance of a diversity system, joint PDF of the combined signal envelopes should be found. Since space diversity system presents the most common way to achieve certain performance improvement, analyses in various environments are reported in the literature. It has been shown that closed form expressions for the joint PDF of two or more random variables (RV) can be found in some cases. In the case of polarization diversity, branch correlation must be taken into account what leads to more complex analytical model, especially when Ricean fading channel is considered [4]. In order to perform complete analytical description of such system, in this paper we propose a transformation that enables decorrelation of signals received over different diversity branches. Further analysis of the system is performed using standard procedure for BER calculation of a diversity system with statistically independent received signals, such as in [5].

BER performance analysis is focused on the polarization diversity system that consists of one transmit antenna and dual polarized receive antenna, as illustrated on Figure 1.

![Fig. 1. Polarization diversity system](image)

Due to several reflections and signal depolarization, signal transmitted from one transmit antenna is received on both, vertically (Vpol) and horizontally polarized (Hpol) antenna, and it can be written as:

\[
r_i(t) = (x_i + jy_i)e^{j\phi_i} + n_i(t), \quad i = 1, 2
\]

(1)

\(r_i(t)\) denotes signal received on Vpol antenna, while \(r_2(t)\) is signal received on the Hpol antenna. \(\phi_\theta(t)\) is the information signal, \(n_i(t)\) denotes zero mean additive white Gaussian noise (AWGN) with variance equal to \(N_0\); \(x_i\) and \(y_i\) are Gaussian random variables with mean values \(m_i\) and \(\mu_i\) and variance \(\sigma_i^2\):

\[
E\{x_i\} = m_i, \quad E\{y_i\} = \mu_i;
\]

\[
E\{x_i^2\} - E^2\{x_i\} = E\{y_i^2\} - E^2\{y_i\} = \sigma_i^2, \quad i = 1, 2
\]

(2)

\(E\{\cdot\}\) denotes mean-value operator.

For the above defined non-zero mean Gaussian RVs, having equal variances, received signal envelopes \(R_i = x_i + jy_i\), \(i = 1, 2\) are distributed according to Ricean distribution [6]. Received signal correlation can be written as:

\[
E\{x_ix_j\} - E\{x_i\}E\{x_j\} = E\{y_iy_j\} - E\{y_i\}E\{y_j\} = \rho \sigma_i \sigma_j
\]

(3)

where \(\rho\) denotes correlation coefficient. Noise envelopes received from the antennas are assumed to be mutually independent, and also independent of the signal components.

\[
E\{n_in_j\} = 0, \quad E\{n_ix_j\} = E\{n_iy_j\} = 0, \quad i, j = 1, 2
\]

(4)

A. Signal Decorrelation

As we already noted, in order to analyze system with a certain level of received envelope correlation, transformation of correlated signals into uncorrelated ones is proposed. Since all signal components are Gaussian, and linear transformation of Gaussian RVs is also Gaussian, the proposed transformation \(T\) is assumed to be linear and defined as:

\[
[\begin{array}{c}
r_{T1}(t) \\
r_{T2}(t)
\end{array}]=\left[\begin{array}{cc}
a_1 & b_1 \\
a_2 & b_2
\end{array}\right]
\]

(5)

\(a_i, b_i, i=1, 2\) are constant matrix elements. Signals \(r_T(t)\), \((i=1, 2)\) present transformed received signals, and are defined as:

\[
r_T(t) = (x_T + jy_T)e^{j\phi_T} + r_T(t), \quad i = 1, 2
\]

(6)

According to the above defined transformation \(T\), signal components \(x_T\) and \(y_T\), as well as noise components \(n_T\), \((i=1, 2)\) are also Gaussian RVs, and can be written as:

\[
\begin{align*}
x_T &= a_1x_1 + b_1x_2; \quad y_T = a_1y_1 + b_1y_2 \\
n_T &= a_1n_1 + b_1n_2; \quad i = 1, 2
\end{align*}
\]

(7)

Mean values and variances of these RVs are:

\[
E\{x_T\} = m_T = a_1m_1 + b_1m_2; \quad E\{y_T\} = \mu_T = a_1\mu_1 + b_1\mu_2 \\
E\{n_T\} = 0; \quad i = 1, 2
\]

(8)

\[
\sigma_T^2 = a_1^2\sigma_1^2 + b_1^2\sigma_2^2 + 2a_1b_1\rho\sigma_1\sigma_2
\]

(9)

Our goal is to find coefficients \(a_i\) and \(b_i\), \(i=1, 2\), such that transformed signals \(r_{T1}(t)\) and \(r_{T2}(t)\) are mutually independent. Thus, coefficients \(a_i\) and \(b_i\) should satisfy the following set of conditions:

1. for transformed noise components to be independent:

\[
a_1a_2 + b_1b_2 = 0
\]

(10)

2. for noise variances to be equal, and have value \(N_0\):

\[
a_1^2 + b_1^2 = 1; \quad \rho = 0
\]

(11)

3. for signal components to be statistically independent:

\[
a_1a_2\sigma_1^2 + b_1b_2\sigma_2^2 = 0
\]

(12)

Solving the equations (10)-(12), transformation matrix \(T\) is obtained in the form:

\[
T = \left[\begin{array}{c}
a_1 \sqrt{1-\rho^2} \\
a_2 \sqrt{1-\rho^2}
\end{array}\right]; \quad \rho = \frac{\left|\frac{E\{x_1y_2\} - E\{x_1\}E\{y_2\}}{E\{x_1^2\} - E\{x_1\}^2}\right|}{\sqrt{\frac{E\{x_1^2\} - E\{x_1\}^2}{E\{x_2^2\} - E\{x_2\}^2}}}
\]

(13)

Thus, parameter \(a\) (as well as transformation matrix \(T\)) is fully determined with Ricean K factors (\(K_1\) and \(K_2\)), correlation coefficient \(\rho\) and cross-polar discrimination \(X\).
It is clear that transformed RVs $x_i$ and $y_j$, $i=1, 2$, are uncorrelated non-zero mean Gaussian RVs having equal variances. Using the fact that two statistically independent Gaussian RVs in quadrature, with different mean values and equal variances, form Ricean distributed RV, it can be concluded that $r_{11}(t)$ and $r_{22}(t)$ present mutually independent signals with envelopes distributed according to Ricean distribution. After the above described transformation, the average SNRs $(\frac{\gamma}{T_i}), i=1, 2, $ and K factors $(K_{T_i}, i=1, 2)$ are related to original parameters according to:

$$\gamma_{T1} = a^2 + (1-a^2)\gamma_{Y1} + 2a\sqrt{1-a^2}\gamma_{Y2}(\rho + \sqrt{K_1K_2})$$

$$\gamma_{T2} = (1-a^2)\gamma_{Y1} + a^2\gamma_{Y2} - 2a\sqrt{1-a^2}\gamma_{Y2}(\frac{1}{1+K_1})\frac{1}{(1+K_2)}(\rho + \sqrt{K_1K_2})$$

$$K_{T1} = \frac{a^2XK_1}{1+K_1} + \frac{(1-a^2)\gamma_{Y1}}{1+K_1} + 2\sqrt{1-a^2}\rho \gamma_{Y2}(\frac{1}{1+K_1})\frac{1}{(1+K_2)}$$

$$K_{T2} = \frac{a^2X}{1+K_2} + \frac{(1-a^2)\gamma_{Y1}}{1+K_2} + 2\sqrt{1-a^2}\rho \gamma_{Y2}(\frac{1}{1+K_1})\frac{1}{(1+K_2)}$$

Now, standard procedure for BER calculation for MRC combining of uncorrelated diversity signals can be applied. Effects of correlation are implied through modification of two parameters describing wireless communication channel, average SNR per diversity branch, given in (14) and Ricean K factor, given in (15).

### B. Bit Error Rate Calculation

Having done the above described decorrelation of the signals received over different diversity branches, BER performance of the analyzed wireless OFDM system is calculated using the MGF (Moment Generating Function) approach. Frequency flat Ricean fading channels are assumed, with the possibility to apply the same expressions for the frequency selective channels in the case of OFDM, as long as the diversity combining is done on a subcarrier basis [7]. Further on, it is considered that the received signals from different diversity branches are subject to the MRC. For the analyzed diversity system with two branches total SNR $(\gamma)$ at the output of the MRC combiner equals the sum of SNRs per diversity branch. After the proposed transformation, system can be described with two independent diversity branches. That means that joint PDF $P(\gamma)$ of the total instantaneous received SNR after MRC can be presented as a product of PDFs of instantaneous SNRs per diversity branch. BER at the output of the wireless OFDM system with the assumed polarization diversity is:

$$P_e = \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \frac{1}{2\pi} \exp \left(-\frac{\gamma}{\sin^2 \theta} \right) \rho(\gamma_{Y1}) \rho(\gamma_{Y2}) \sin \theta d\gamma_{Y1} d\gamma_{Y2} d\theta$$

Changing the order of integration, and having in mind that MGF is defined as Laplace transformation of PDF, (16) can be finally written as:

$$P_e = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \frac{1}{2\pi} \exp \left(-\frac{1}{\sin^2 \theta} \right) M_{\gamma_{Y1}}(s) M_{\gamma_{Y2}}(s) d\theta$$

In Ricean fading case, MGF is of the following form [5]:

$$M_{\gamma_{Y1}}(s) = \frac{1+K_{T1}}{1+K_{T1}-s\gamma_{T1}} \exp \left(-\frac{K_{T1}}{1+K_{T1}-s\gamma_{T1}} \right)$$

Thus, BER expression is finally obtained in a form of single, finite range integral given in (17) that can be calculated using standard mathematical packages.

### III. RESULTS

In this paper we analyze the influence of two main parameters: correlation coefficient and XPD on diversity gain of the OFDM system with implemented polarization diversity. For that purpose correlation coefficient is assumed to be 0 (ideal case of perfect decorrelation), 0.3 (typical values vary from 0.2-0.4 [3]) and 0.7, that is considered as maximal value of correlation coefficient for which effective diversity gain can be achieved [2].

XPD is more dependent on environment than correlation coefficient, therefore having larger influence on overall system performance. Different values are considered, from 0 dB (ideal case) to 10 dB (typical values in urban environments).

BER performance of the described polarization diversity system is shown in Figure 2. BER curve for the system with no diversity is also presented, in order to determine diversity gain obtained by implementing dual polarized receive antenna.

![Fig. 2. Impact of correlation coefficient and XPD on BER performance](image-url)
receiving end, comparison with the simulation results is performed. Simulation is carried in the baseband, assuming Ricean fading channel, slow enough that channel parameters are constant during at least one OFDM symbol duration. Guard interval is chosen to be longer than channel impulse response, in order to eliminate ISI. Perfect channel knowledge is assumed, as well as perfect synchronization between the transmitter and the receiver. Since the considered diversity system consists of only one transmit antenna, it is clear that crosstalk signal received at the Hpol antenna results only from reflections. In that case, Ricean $K$ factor is assumed to be lower than $K$ factor of copolarized signal [8]. Some of the used simulation parameters are given in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data modulation/data rate</td>
<td>BPSK/ 50 Mbps</td>
</tr>
<tr>
<td>Number of subcarriers</td>
<td>256</td>
</tr>
<tr>
<td>Max delay spread</td>
<td>200 ns</td>
</tr>
<tr>
<td>OFDM symbol duration</td>
<td>6.144 µs</td>
</tr>
<tr>
<td>Correlation coefficient</td>
<td>0.3</td>
</tr>
<tr>
<td>Ricean K factors</td>
<td>$K_1=4$ dB, $K_2=0$ dB</td>
</tr>
<tr>
<td>XPD</td>
<td>0 dB, 6 dB, 10 dB</td>
</tr>
</tbody>
</table>

Figure 3 gives the BER values obtained through the simulation. For the sake of direct comparison, analytically calculated BER values are also presented in the same figure as well as BER curve for the system with no diversity. Although both parameters: correlation coefficient and XPD have significant influence on BER, for the sake of figure clarity, the comparison between analytical and simulation results is presented only for one fixed value $\rho=0.3$.

Fig. 3. Comparison of analytical and simulation BER results for OFDM polarization diversity system.

The comparison of calculated and simulated results prove that the proposed analytical approach, which enables transformation of the received signals into uncorrelated ones while their correlation is transformed into specific propagation parameters, can be considered as valid. Using the described model, the level of the considered system performance enhancements can be precisely determined. That offers important possibilities for the future research work when this type of combined OFDM-diversity technique is considered.

IV. CONCLUSION

It is well known that diversity techniques are relatively simple and effective solutions for improving system performance in the multipath fading environment. For the chosen wireless OFDM systems, the work presented in this paper shows that polarization diversity can be considered as an alternative to commonly used space diversity, especially in urban environments with the lack of space to mount two or more spatially separated antennas. In such environments multiple reflections of the transmitted signal lead to certain depolarization of the transmitted wave, i.e. depolarization mechanisms provide orthogonally polarized signal, i.e. additional diversity branch, even in the case when there is only one polarization at the transmitter.

In this paper we develop complete analytical model for the OFDM polarization diversity system output BER calculations when using one vertically polarized transmit antenna and dual polarized receive antenna, considering Ricean fading environment. The described method is based on the proposed linear transformation which transforms correlated signals into uncorrelated ones. Further analysis implies BER calculation procedure applied to uncorrelated signals, using modified system parameters: the average received signal–to–noise ratio (SNR) per diversity branch and Ricean $K$ factor. The validity of the presented model is proved using the simulation of the considered system. The obtained results show that, despite a certain level of the correlation between the signals from different diversity branches and power imbalance between them, polarization diversity system provides significant OFDM performance improvements. Since it utilizes only one, dual polarized antenna, it is space and cost effective solution interesting to both, network operators and manufacturers of mobile equipment and terminals.

REFERENCES