Fishway Optimization Revisited

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Abstract River fishways are hydraulic structures enabling fish to overcome stream obstructions (for instance, dams in hydroelectric power plants). This paper presents a combination of mathematical modelling and optimal control theory in order to improve the optimal shape design of a fishway. The problem can be formulated within the framework of the optimal control of partial differential equations, approximated by a discrete optimization problem, and solved by using a gradient-type method (the Spectral Projected-Gradient algorithm). Numerical results are shown for a standard real-world situation.

1 Introduction

Several species of fish (salmon, trout, carp, sturgeon, eel...) attempt migrations between fresh and salt water on a regular basis, on time scales varying from daily to annual, and with distances ranging from a few meters to thousands of kilometers. So, when we construct an artificial barrier in a river (for example, a weir or a dam

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in a hydroelectric power plant) European legal regulations force us to also install a fishway in order to allow fish to overcome it.

Fishways are hydraulic structures placed around man-made barriers to assist the natural migration of fish. The vertical slot fishway is the more widely adopted for upstream passage of fish in stream obstructions. It consists of a rectangular channel with a sloping floor that is divided into a reduced number of pools (see Fig. 1). Water runs downstream in this channel, through a series of vertical slots from one pool to the next one below. The water flow forms a jet at the slot, and the energy is dissipated by mixing in the pool. Fish ascends, using its burst speed, to get past the slot and then it rests in the pool until the next slot is tried [4].

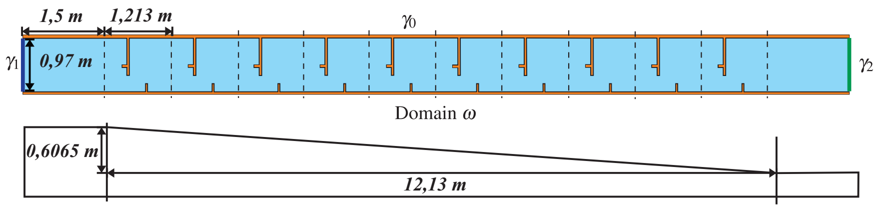

Fig. 1 Ground plant (domain ω with lateral boundary γ_0, inflow boundary γ_1 and outflow boundary γ_2) and elevation of a standard fishway

During recent decades much attention has been paid, both from the theoretical and the experimental viewpoint, to the hydraulic characteristics, the flow regimes, and the vorticity structures in all types of river fishways. Nevertheless, the fundamental role of a correct design in the fishway has been much less studied. As far as we know, the optimal design of a vertical slot fishway has only been previously analyzed by the authors [1, 2] in a simple case.

As stated above, the objective of a fishway is enabling fish to overcome obstructions. In order to get it, water velocity in the fishway must be controlled. Specifically, this means that in the zone of the channel near the slots, the velocity must be close to a desired velocity suitable for fish leaping and swimming capabilities. In the remains of the fishway, the velocity must be close to zero for making possible the rest of the fish. Moreover, in all the channel, flow vorticity must be minimized.

Water velocity can be directly controlled by determining the optimal shape of the fishway, that is, the location and length of the baffles separating the pools. In this work we use mathematical modelling and optimal control theory to address the optimal design of a fishway. In order to do this, we begin presenting a mathematical model (shallow water equations) to simulate the water velocity in a fishway and giving a mathematical expression to evaluate the quality of that velocity field in terms of the fish capabilities. Next, we study the problem of the optimal design of a fishway. We describe the problem, formulate it as a shape optimization problem, and show that it can be approximated by a discrete optimization problem. A gradient-type method (the Spectral Projected-Gradient algorithm) is proposed to solve this problem, and numerical experiences are reported.

2 Simulation of Fishway Hydrodynamics

Let $\omega \subset \mathbb{R}^2$ be the ground plant of a fishway consisting of a rectangular channel divided into a small number of pools with baffles and sloping floor, and two transition pools (one at the beginning and another one at the end of the channel) with no baffles and flat floor. A scheme of the standard fishway used in this paper can be seen in Fig. 1: water enters by the left side and runs downstream to the right side, while fish ascend in the opposite direction [1]. The number of pools (ten) and the dimensions of the full channel correspond to an experimental scale fishway reported by Puertas et al. [5].

Water flow inside the domain ω along the time interval $(0,T)$ is governed by the (Saint-Venant) shallow water equations [2]:

$$\frac{\partial H}{\partial t} + \nabla.\mathbf{Q} = 0, \quad \frac{\partial \mathbf{Q}}{\partial t} + \nabla.(\frac{\mathbf{Q}}{H} \otimes \mathbf{Q}) + gH\nabla(H-\eta) = \mathbf{f} \quad \text{in } \omega \times (0,T), \tag{1}$$

where $H(x,y,t)$ is the height of water at point $(x,y) \in \omega$ and at time $t \in (0,T)$, $\mathbf{u}(x,y,t) = (u,v)$ is the depth-averaged horizontal velocity of water, $\mathbf{Q}(x,y,t) = \mathbf{u}H$ is the flow per unit depth, $\eta(x,y)$ represents the bottom geometry, and second member $\mathbf{f}$ collects all the effects of bottom friction, atmospheric pressure and so on. These equations must be completed with a set of initial and boundary conditions on the lateral boundary of the channel (denoted by γ_0), the inflow boundary (denoted by γ_1), and the outflow boundary (denoted by γ_2).

By using these notations we can give a mathematical expression to evaluate the quality of water velocity in the fishway. We have to bear in mind two objectives:

(i) In the zone of the channel near the slots (say the lower third) the velocity must be as close as possible to a typical horizontal velocity σ suitable for fish leaping and swimming capabilities; in the remaining of the fishway, the velocity must be close to zero for making possible the rest of the fish. In short, the velocity must be close to the following target velocity:

$$\mathbf{v}(x_1,x_2) = \begin{cases} (\sigma,0), & \text{if } x_2 \leq \frac{1}{3}W, \\ (0,0), & \text{otherwise }, \end{cases} \tag{2}$$

where W is the width of the channel (in our case $W = 0.97\,m$, as shown in Fig. 1).

(ii) Flow vorticity must be minimized in all the channel in order to avoid fish disorientation, that is, the vorticity must be as much reduced as possible.

According to this, if we fix a weight parameter $\alpha \geq 0$ for the role of the vorticity and define the objective function

$$J = \frac{1}{2}\int_0^T \int_\omega \|\frac{\mathbf{Q}}{H} - \mathbf{v}\|^2 + \frac{\alpha}{2}\int_0^T \int_\omega |curl(\frac{\mathbf{Q}}{H})|^2, \tag{3}$$

the water velocity $\mathbf{u} = \mathbf{Q}/H$ is better for our purposes as the value of the cost function J becomes smaller.

In order to evaluate J, we need to solve the shallow water equations (1). In this work we use an implicit discretization in time, upwinding the convective term by the method of characteristics, and Raviart-Thomas finite elements for the space discretization [1]. If we consider a time step $\Delta t = T/N > 0$ and a Lagrange-Galerkin finite element triangulation τ_h of the domain ω, the numerical scheme provides an approximated flux $\mathbf{Q}_h^k$ and an approximated height H_h^k, which are piecewise-linear polynomials and discontinuous piecewise-constant functions, respectively, and are required to compute an approximated value of J.

3 Optimal Shape Design of a Fishway

In this section we study how to improve the optimal design of a fishway. As commented before, we can control the water velocity through the location and length of the baffles in the pools. To fix ideas, we take the channel described in previous section, we assume that the structure of the ten pools with sloping floor has to be the same (the shape of the complete fishway is given by the shape of the first pool) and then, we take the three midpoints corresponding to the end of the baffles in the first pool (points $a = (s_1, s_2)$, $b = (s_3, s_4)$ and $c = (s_5, s_6)$ in Fig. 2) as design variables.

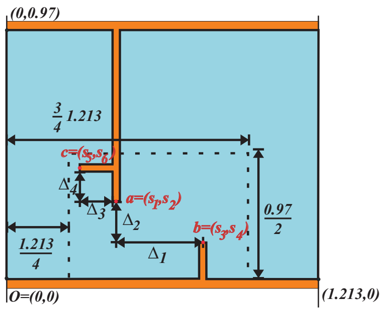

Fig. 2 Scheme of the first pool

So, we look for points a, b and c providing the best velocity for fish (i.e. minimizing the function J given by (3)), but, previously, we must impose several design constraints on these points. First, we assume that points a, b and c are inside the dashed rectangle of Fig. 2, that is, the following twelve geometrical relations must be satisfied (in order to avoid unnecessary symmetrical duplicate solutions):

$$\frac{1}{4}1.213 \leq s_1, s_3, s_5 \leq \frac{3}{4}1.213, \quad 0 \leq s_2, s_4, s_6 \leq \frac{1}{2}0.97. \tag{4}$$

A second type of constraints is related to the fact that the vertical slot must be large enough so that fish can pass comfortably through it. This translates into the two following linear constraints:

$$\Delta_1 = s_3 - s_1 \geq 0.1, \quad \Delta_2 = s_2 - s_4 \geq 0.05. \tag{5}$$

Finally, a third type of constraints is related to structural stability, as given by the two additional linear constraints, with $r = 0.0305$ the half width of the baffle:

$$\Delta_3 = s_1 - s_5 \geq \frac{1}{2}0.0305, \quad \Delta_4 = s_6 - s_2 \geq \frac{1}{2}0.0305. \tag{6}$$

Then, the optimization problem can be formulated as follows:
Problem ($\mathscr{P}$): Find the optimal shape of domain ω, that is, find $s = (a,b,c) = (s_1,s_2,s_3,s_4,s_5,s_6) \in \mathbb{R}^6$ verifying constraints (4)-(6), in such a way that $\mathbf{Q}$ and H, given by the solution of the state system (1) on the fishway $\omega \equiv \omega(s)$, minimize the objective function $J \equiv J(s)$ defined by (3). (A mathematical analysis of a simpler related problem can be found in Alvarez-Vázquez et al. [1].)

For its numerical resolution we propose the Spectral Projected-Gradient (SPG) algorithm, due to Birgin et al. [3], where the required derivatives are approached by finite difference approximations. This algorithm provides in each iteration an admissible point in the convex set Ω defined by the constraints (4)-(6). To achieve this, we designed a special polynomial-time algorithm to compute the projection of a vector over this set. Moreover, global convergence for the SPG method is assured under reasonable hypotheses (see [3] for more details).

4 Numerical Results

We give here some numerical results obtained for a standard situation. We considered the fishway under study, whose scheme is shown in Fig. 1. The time interval for the simulation was $T = 300s$. Moreover, for the sake of simplicity, for the second member $\mathbf{f}$ we considered only the bottom friction stress for a Chezy coefficient of $57.36m^{0.5}s^{-1}$. For the objective function we took a target velocity value $\sigma = 0.8ms^{-1}$, and a weighting parameter $\alpha = 0.15$. Finally, for the time discretization we took $N = 3000$ (that is, a time step of $\Delta t = 0.1\,s$) and, for the different space discretizations, we tried several regular triangulations of about 9500 elements.

Although we have developed many numerical experiences, we present here only one example for this realistic problem. So, applying the Spectral Projected-Gradient algorithm, we have passed, after only 6 iterations, from the initial cost $J = 551.05$, corresponding to the random points $a = (0.6, 0.15)$, $b = (0.9, 0.07)$, $c = (0.5, 0.4)$, to the minimum cost $J^* = 242.67$, corresponding to the optimal design variables $a^* = (0.71, 0.16)$, $b^* = (0.91, 0.06)$, $c^* = (0.41, 0.42)$. The total process took about 38 hours of CPU time in a laptop with Intel Pentium 4 microprocessors.

Two close-ups of the central pool are shown, respectively, in Figs. 3 and 4. In the case of the initial random shape (Fig. 3) we can identify the standard flow patterns presented, for instance, in [5]: a direct flow region where the flow circulates in a curved trajectory at high velocity from one slot to the next downstream, and two recirculation regions - the larger one located between the long baffles and the smaller one located between the short baffles - flowing in opposite directions. In the case of the optimal shape (Fig. 4) the direct flow velocity is very close to the target horizontal velocity **v**, the smaller recirculation region is completely removed, and the larger one is highly reduced.

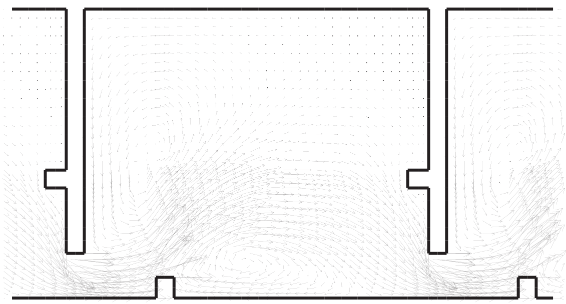

Fig. 3 Initial random horizontal velocity field in the central pool at final time $T = 300\,s$

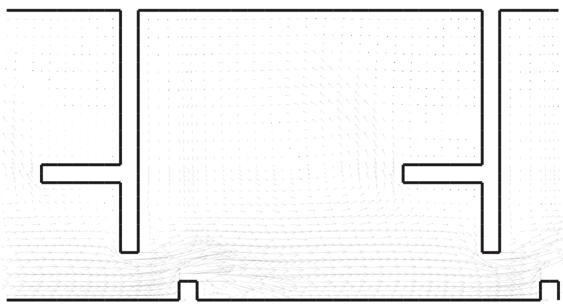

Fig. 4 Optimal horizontal velocity field in the central pool at final time $T = 300\,s$

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