Economic Design of Water Distribution Systems in Buildings

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Abstract

This paper discusses an engineering optimization problem which arises in hydraulics and is related to the use of a new criterion for sizing water distribution piping in large buildings. The optimization model aims to find the most suitable interior pipe diameters for the various pipes in the system, using commercial sizes and minimizing the overall installation cost according to some boundary conditions. The problem is formulated as a nonconvex nonlinear program and a branch-and-bound algorithm is introduced for its solution. A procedure is proposed to obtain a feasible solution with standard values from the optimal solution of the nonconvex program. The performance of the algorithm is analysed for a real-life problem and the cost of the computed solution is assessed, showing the appropriateness of the model and the optimization techniques.

Keywords: Nonlinear Optimization, Global Optimization, Hydraulics.

1 Introduction

If its design flows and geometry are known, the sizing of a water pipe system in a building can be found by calculating the most suitable diameters for the various pipes in order to satisfy the boundary conditions and some constraints related to velocities and pressures. The sizing of a water piping system is usually based on one of the following simplified criteria (Silva-Afonso, 2001; Brater, 1976):

i. the maximum permitted velocity criterion,

ii. the maximum total permitted head loss criterion.

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The first criterion leads to a cheaper solution and it is the best option, if practicable. If the design flows in each pipe are known, the diameters are computed in order to minimize pipe size and satisfy some limits for velocity. But this criterion usually means considerable head losses in the water distribution system, and therefore requires sufficient head to guarantee the minimum residual pressures in the various fittings and fixtures.

The second criterion should be used when residual pressures are low, or when the total head losses in some circuits within the water distribution system are too high (critical circuits).

The application of this second sizing criterion is only justified in critical circuits where residual pressures or pressure fluctuations are in question. It is therefore important, for reasons of cost, to employ the first criterion in the other circuits.

This paper proposes a new criterion for the sizing of water distribution piping in buildings; it is called the economic design criterion (Silva-Afonso, 2001). Although similar in some respects to the second criterion, it is justified by the fact that the computation based on a medium value of the unit friction loss does not guarantee a solution that is optimal for the overall water distribution system cost.

This new cost design criterion aims to compute the interior piping diameters to be used in the various pipes of the circuit. These must belong to a finite collection of standard values corresponding to a range of commercially available diameters, so that the imposed boundary conditions are satisfied, together with some constraints related to velocities and head losses within the pipes, and the global installation cost is minimized. Therefore, the decision variables of the corresponding optimization problem must be the interior piping diameters of the various pipes in the circuit. Other parameters, such as the geometric features of the circuit, minimum residual pressures in the fixtures and pipe material are normally considered to be problem data.

The total head loss should neither exceed the amount of the available head, nor the value of the maximum established pressure fluctuations.

In this paper an optimization model representing this new criterion is described. The formulation of this model involves a global optimization problem consisting of minimizing a nonlinear nonconvex function on a convex set defined by inequality constraints. A branch-and-bound algorithm is proposed for finding an optimal solution to this nonconvex program. The algorithm is based on underestimating functions to provide lower and upper bounds that reduce the overall search in the tree. In the optimization problem to be solved, the diameter variables should belong to a discrete set of standard values. A crash procedure is described to obtain a good feasible solution for this optimization problem from the global minimum of the associated nonconvex program. Computational experience is included to show that the branch-and-bound algorithm performs quite well for the solution of a real-life model. Furthermore the crash procedure is able to find a feasible solution which corresponds to the objective of the model.

The model was tested for a large public building, and led to a better solution than the one obtained by the traditional sizing procedures.

The structure of the paper is as follows. In Section 2 the formulation of the model is introduced. Section 3 includes some properties of the associated optimization problem. The branch-and-bound algorithm for processing the nonconvex program is described in Section 4. Section 5 is devoted to the procedure for obtaining a good feasible solution with standard values from the optimal solution of the original problem. Computational experience with a real-life instance of the proposed problem is reported in Section 6. Finally some conclusions and some ideas for future research are presented in the last section.

2 Model Description

In hydraulics, the equation of continuity and the Bernoulli equation are usually applied by considering that the flow does not change with time, the fluid is incompressible and the pressure distribution in the cross-sections is hydrostatic. In these circumstances, it is easy to find some useful relationships among the different variables that allow the solution of the hydraulic design problem (Silva-Afonso, 2001).

The energy between flow sections is found by the Bernoulli theorem, which expresses the hydraulics of the Energy Conservation Principle (Silva-Afonso, 2001).

In the case of real fluids in permanent motion, the total energy or total head H decreases along the trajectory, due to the work done by the forces resisting the motion, arising from the interaction between the fluid motion and the walls of the pipe. The decrease of the energy line per unit of length is equal to the work done by the resistant forces, per unit of liquid weight and per unit of length. It is usually designated by head loss per unit of length or unitary friction loss, and denoted by J (Smith, 1994; Walski, 1990; Wise, 1986). These head losses can be computed, for example, by the Flamant Formula (Silva-Afonso, 2001).

In practice, the value of J does not change during flow assuming that the flow is uniform in terms of motion and according to the characteristics of the resistant environment. Therefore, in water piping systems, the total continuous head losses along the pipes, ΔH_i, in each pipe i are calculated as the product of the unit friction loss J_i by the length of the pipe, L_i. Head losses can also occur in singularities such as piping fittings (curves and reductions, for instance), pipe accessories (valves and meters) and in equipments (treatment, heating, etc). Head losses in singularities are designated by local and are represented by ΔH_L.

In water distribution piping in buildings, local head losses cannot be ignored, as they have a significant magnitude when compared with continuous head losses. However, individual computation of local head losses for most pipe fittings and accessories with low head losses is not justified. But they can be computed as an equivalent piping length that produces equal head loss. In practice, real lengths of the circuits are bounded in a percentage (ρ) between 15 and 25 %, depending on the material. Piping accessories and equipment producing very significant local head losses (globe valves, etc.) are not included in this procedure.

The main components of a water piping system are the pipes (including fittings), the piping accessories (valves and meters) and equipment for treatment, heating, etc. In order to apply the economic sizing criterion mentioned in the previous section, it is necessary to define cost functions relating unit costs to interior diameters for all the parts whose characteristics depend on the values attributed to the decision variables.

As the characteristics of the meters and the treatment and heating equipment only depend on the water flow, cost functions have to be established for any pipes and valves that may be installed in the circuits. The unit cost related to the installed pipes and valves must be global, i.e. they must take into account all the accessories (connection and fitting parts), labour (cost per worker and any complementary works) and charges (administration and tax charges) needed for the installation.

(i) Objective Function

The pipe cost function is usually expressed as a polynomial of degree two

$$CC = -a\,x^2 + b\,x - c \tag{1}$$

where CC is the unit cost of the installed pipes, in €/meter, x is the interior pipe diameter, in meters, and a, b and c are previously known positive constants (Silva-Afonso, 2001). The cost function of the valves is given as a polynomial of degree three

$$CV = -\alpha\,x^3 + \beta\,x^2 - \gamma\,x + \eta \tag{2}$$

where CV is the cost of the installed valve, in €, α, β, γ, η are previously known positive constants and x is the interior pipe diameter, in meters (Silva-Afonso, 2001). These expressions imply that the objective function for a circuit with n pipes and s valves takes the form

$$CT = \sum_{k=1}^{n} CT_k + \sum_{j=1}^{s} CV_j = \sum_{k=1}^{n} (\,-a_k\,x_k^2 + b_k\,x_k - c_k\,)\,L_k + \sum_{j=1}^{s} v_j\,(\,-\alpha_j\,x_j^3 + \beta_j\,x_j^2 - \gamma_j\,x_j + \eta_j\,) \tag{3}$$

where $CT_k = CC_k\,L_k$ represents the installed piping cost in the pipe k, CC_k and CV_j are the installed pipe unit cost in the pipe k, in €/meter, and the cost of the installed valve j, in €, L_k is the length of the pipe k, x_k is the interior pipe diameter in the pipe k, x_j is the interior diameter of the valve j, in meters, v_j is the number of valves in the pipe j and $s \leq n$.

(ii) Bounds and other Constraints

The velocity bounds are a consequence of the regulations, which require the velocity in each pipe of the circuit not to exceed the minimum and maximum fixed values (normally expressed in m/s).

By expressing those bounds as functions of the diameters, with the design flow Q normally in m^3/s and x in meters, it is obtained, for a circuit with n pipes,

$$x_k \leq \varphi_k Q_k^{0.5} = u_k \tag{4}$$

and

$$x_k \geq \psi_k Q_k^{0.5} \tag{5}$$

respectively (Q_k represents the design flow in pipe k), where φ_k and ψ_k are positive constants, for $k = 1, 2, ..., n$.

According to Portuguese regulations, which are similar to other European regulations, the design flow velocities in each pipe of any circuit must not to exceed the minimum and maximum bounds of $0.5\,m/s$ and $2.0\,m/s$, respectively. With respect to these velocities, it has $\psi_k = 0.798$ and $\varphi_k = 1.596$, for each $k = 1, 2, ..., n$.

Additional bounds are usually considered in some pipes in the circuit, for reasons of acoustic comfort (different from the regulation velocities). For a high comfort level, the additional bound

$$x_k \geq 0.400\,Q_k^{0.37}\,,\ k = 1, 2, ..., n \tag{6}$$

can also be included in the model. If bounds (5) and (6) are held to be simultaneously satisfied, then the lower bound l_k of each variable x_k should be given by

$$l_k = max\left\{0.798\,Q_k^{0.5}\,,\, 0.400\,Q_k^{0.37}\right\}$$

The Flamant Formula is used to establish the constraint associated to head losses. It is considered jointly with the percentage ρ of the real length of the circuit upper bound (Silva-Afonso, 2001). Thus, in each pipe k, with ΔH_k in meters, Q_k in m^3/s and x_k in meters, it is obtained

$$\Delta H_k = (1 + 0.01 \times \rho)\,L_k\,h_k\,Q_k^{1.75}\,x_k^{-4.75}\,,\ k = 1, 2, ..., n \tag{7}$$

where $1+0.01\times\rho$ represents the increase factor applied to the real pipe lengths (Silva-Afonso, 2001), L_k represents the length of the pipe k and h_k are previously known positive constants.

For pipes k with no valves, the total head loss should not exceed the maximum permitted head loss ΔH_M. Therefore the following constraint has to be considered

$$\sum_{k=1}^{n}\left\{(1 + 0.01 \times \rho)\,L_k\,h_k\,Q_k^{1.75}\,x_k^{-4.75}\right\} \leq \Delta H_M \tag{8}$$

If head losses in valves are significant (globe valves, for example, present high head losses, even in a total outlet position), the constraint (8) should be modified. In fact, local head losses should be considered individually, with direct computation of the head loss or by using an equivalent virtual length

$$L_e = \frac{K}{\lambda}\,x \tag{9}$$

where K represents the local head loss coefficient and λ is the resistance coefficient for each valve. It is assumed here that for a given material, fluid and temperature, the relation $\dfrac{K}{\lambda}$ may be considered practically unchanged, that is,

$$L_e = C\, x$$

where C is a positive constant. Constraint (8), relating to total head loss, can thus be replaced by

$$\sum_{k=1}^{n} h_k\, Q_k^{1.75}\, x_k^{-4.75}\, \{\, (\, 1 + 0.01 \times \rho\,)\, L_k + v_k\, C\, x_k \,\} \,\leq\, \Delta H_M \tag{10}$$

where v_k represents the number of valves to be installed in pipe k, $k = 1, 2, ..., n$.

All these considerations lead to an optimization model that is described next. The data for this model is as follows:

$T = \{\, 1, 2, ..., n\, \}$: set of the pipes in the circuit ($n = |T|$ represents the total number of pipes in the circuit);

$V = \{\, 1, 2, ..., s\, \}$: set of the pipes in the circuit where the installation of valves is considered ($V \subseteq T$);

S : set of the pipes in the circuit where it is required to equate the local head losses in valves ($S \subseteq V \subseteq T$);

L_k : length of pipe k, $k \in T$ ($L_k > 0\, , \forall\, k \in T$);

Q_k : design flow in pipe k, $k \in T$;

v_k : number of valves to be installed in pipe k, $k \in T$ ($v_k > 0$ if $k \in V$ and $v_k = 0$ if $k \in T \setminus V$);

C : positive constant;

ρ : constant corresponding to the percentage of an upper bound to the real length of the circuit, taking into account the compensation for the various local head losses in pipe fittings ($0 < \rho \leq 100$);

The decision variables x_k of the model correspond to the interior pipe diameter in pipe k, $k \in T$ ($x_k \geq 0\, , \forall\, k \in T$). The optimization model takes the form

NLP: Minimize $$f(x) = \sum_{k=1}^{n} f_k(x_k) \tag{11}$$

subject to $$\sum_{k=1}^{n} g_k(x_k) \leq g \tag{12}$$

$$l_k \leq x_k \leq u_k\, , \; k = 1, 2, ..., n \tag{13}$$

where

$$f_k(x_k) = L_k\,(\,-a_k\,x_k^2\,+\,b_k\,x_k\,-\,c_k\,)\,+\,v_k\,(\,-\alpha_k\,x_k^3\,+\,\beta_k\,x_k^2\,-\,\gamma_k\,x_k\,+\,\eta_k\,) \quad (14)$$

$$v_k = \begin{cases} > 0\,, & \text{if } k\,=\,1, 2, ..., \,s \\ \\ = 0\,, & \text{if } k\,=\,s+1, ..., \,n \end{cases} \quad (15)$$

$$g_k(x_k) = d_k\,x_k^{-4.75}\,(\,e_k\,+\,q_k\,x_k\,) \quad (16)$$

Furthermore $s \leq n$, L_k, a_k, b_k, c_k, α_k, β_k, γ_k, η_k and g are given positive constants and

$$d_k\;=\;h_k\,Q_k^{1.75}\;>\;0\;;$$

$$e_k\;=\;(\,1+0.01\times\rho\,)\,L_k\;>\;0\;;$$

$$q_k\;=\;C\,v_k\;\geq\;0$$

for all $k = 1, \ldots, n$.

It should be added that a global minimum for this optimization model is not, in principle, feasible for the problem, as the diameters ought to belong to a discrete collection of previously known standard values corresponding to the range of commercially available diameters . As discussed in the next sections, the optimal values obtained for the decision variables for the range of commercial diameters should be approximated in order to get a good acceptable solution for the hydraulics model.

3 Properties of the Optimization Problem

In this section, some properties of the objective function and of the feasible set of the nonlinear program NLP presented in the preceding section are investigated. The following properties taken from Bazaraa et al. (1993); Martos (1975) are useful in this context.

Property 1. *The sum of strictly convex functions on a convex set $K \subseteq \mathbb{R}^n$ is a strictly convex function on K.*

Property 2. *If f is a convex function on a convex set $X \subseteq \mathbb{R}^n$ and $\theta \in \mathbb{R}$, then*

$$K = \{x \in X : f(x) \leq \theta\}$$

is a convex set.

Property 3. *If g is a real function of one real variable twice continuously differentiable on a interval I, with $g''(x) > 0\,(<0)$ for all interior points x of I, then g is strictly convex (concave) on I.*

It then follows from this last result that

Property 4. *If $\theta > 0$ and $\beta \notin [0, 1]$, then the function*

$$\begin{array}{rccl} g : & \mathbb{R}_+ & \longrightarrow & \mathbb{R} \\ & x & \longmapsto & \theta\, x^{\beta} \end{array}$$

is strictly convex on $\mathbb{R}_+ = \{x \in \mathbb{R} : x > 0\}$.

It can then be established the following two main results:

Theorem 1. *The feasible set of the program NLP is a convex set.*

Proof: By properties 1 and 4, the functions g_k given by (16) are strictly convex on $\mathbb{R}_+$, for all $k = 1, 2, ..., n$. Since $g(x) = \sum_{k=1}^{n} g_k(x_k)$, then the feasible set of the program is defined by

$$\mathrm{K} = \{\, x \in \mathbb{R}^n : \, g(x) \leq g\,;\, l_k \leq x_k \leq u_k\,,\; k = 1, 2, ..., n \,\}$$

Then K is a convex set, by properties 1 and 2. □

Theorem 2.

(i) *If $v_k = 0$, then f_k is a strictly concave function on $[\, l_k\,,\, u_k\,]$.*

(ii) *If $\overline{x}_k = \dfrac{v_k\,\beta_k - L_k\,a_k}{3\,\alpha_k\,v_k}$, then*

- **(a)** *$\overline{x}_k \leq l_k \Rightarrow f_k$ is strictly concave on $[\, l_k\,,\, u_k\,]$;*
- **(b)** *$\overline{x}_k \geq u_k \Rightarrow f_k$ is strictly convex on $[\, l_k\,,\, u_k\,]$;*
- **(c)** *$l_k < \overline{x}_k < u_k \Rightarrow f_k$ is strictly convex on $[\, l_k\,,\, \overline{x}_k\,]$ and strictly concave on $[\,\overline{x}_k\,,\, u_k\,]$.*

Proof: Since

$$f_k(x_k) = L_k\,(\,-a_k\,x_k^2 \,+\, b_k\,x_k \,-\, c_k\,) \,+\, v_k\,(\,-\alpha_k\,x_k^3 \,+\, \beta_k\,x_k^2 \,-\, \gamma_k\,x_k \,+\, \eta_k\,)$$

then for each $x_k \in \mathbb{R}^1$,

$$f_k''(x_k) = 2\;[(v_k\,\beta_k - L_k\,a_k) - 3\,\alpha_k\,v_k\,x_k]$$

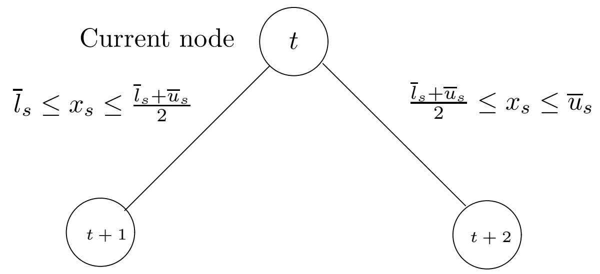
The result now follows from property 3. □

4 Branch-and-Bound Algorithm for the Optimization Problem

A global optimal solution of the program NLP may be found by using a branch-and-bound algorithm based on a decomposition of the feasible set of the program, similar to that described in Horst et al. (2000); Horst and Tuy (1993); Konno and Wijayanayake (2001). This procedure exploits a binary tree, where each node is associated to a Nonlinear Program of the form

$$\begin{aligned} \textbf{NLP(node):} \quad \text{Minimize} \quad & f(x) = \sum_{k=1}^{n} f_k(x_k) \\ \text{subject to} \quad & \sum_{k=1}^{n} g_k(x_k) \leq g \\ & \overline{l}_k \leq x_k \leq \overline{u}_k, \;\; k = 1, 2, \ldots, n \end{aligned}$$

with $l_k \leq \overline{l}_k < \overline{u}_k \leq u_k$, for all $k = 1, \ldots, n$. In order to explain the branching technique, let t be a node of the binary tree associated to the Nonlinear Program. For a variable x_s, consider the partition of $[\overline{l}_s, \overline{u}_s]$ in two intervals $[\overline{l}_s, \frac{\overline{l}_s+\overline{u}_s}{2}]$ and $[\frac{\overline{l}_s+\overline{u}_s}{2}, \overline{u}_s]$. Then two daughter nodes can be generated from the current node according to the following scheme:



The NLPs associated with the newly generated nodes $(t+1)$ and $(t+2)$ are obtained from the previous one by replacing the bound constraints $\overline{l}_s \leq x_s \leq \overline{u}_s$ by the corresponding constraints

$$\overline{l}_s \leq x_s \leq \frac{\overline{l}_s + \overline{u}_s}{2} \quad \text{and} \quad \frac{\overline{l}_s + \overline{u}_s}{2} \leq x_s \leq \overline{u}_s$$

As is discussed in Konno and Wijayanayake (2001), for this method to be efficient, techniques for finding lower and upper bounds have to be developed. If for a given node the current lower bound was greater than or equal to the best upper bound, then there is no need to search from this node.

Lower bounds are computed by constructing an underestimating function

$$h(x) = \sum_{k=1}^{n} h_k(x_k)$$

where, for each $k = 1, 2, ..., n$, h_k is convex on $\left[\overline{l}_k, \overline{u}_k\right]$ and $h_k(x_k) \leq f_k(x_k)$, for all $\overline{l}_k \leq x_k \leq \overline{u}_k$.

As discussed in the previous section, if is assumed that for each $k = 1, 2, ..., n$, $f_k''(x_k)$ does not vanish in the interval $\left[\overline{l}_k, \overline{u}_k\right]$, then there are two possible cases:

Case 1 - f_k is convex on $\left[\,\bar{l}_k\,,\,\overline{u}_k\,\right]$. Then $h_k(x_k) = f_k(x_k)$, for all $\bar{l}_k \le x_k \le \overline{u}_k$.

Case 2 - f_k is concave on $\left[\,\bar{l}_k\,,\,\overline{u}_k\,\right]$. Consider the linear approximation $h_k(x_k) = r_k\,x_k + s_k$ of f_k in $\left[\,\bar{l}_k\,,\,\overline{u}_k\,\right]$ such that $f_k(x_k) > r_k\,x_k + s_k$, for all $\bar{l}_k < x_k < \overline{u}_k$, as illustrated in Figure 1.

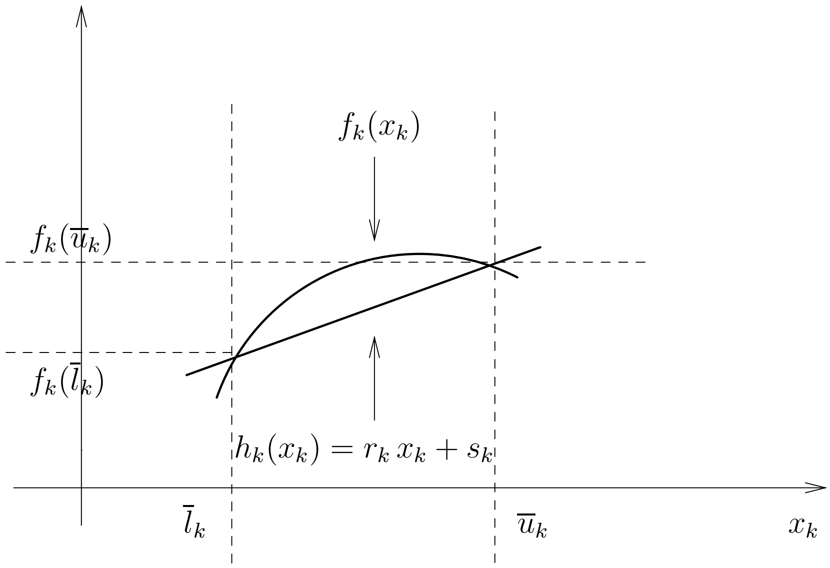


Figure 1: Linear Approximation

Furthermore, it is easy to show that the values of r_k and s_k are as follows

$$r_k = \begin{cases} 0 & \text{if } f_k(\overline{u}_k) = f_k(\bar{l}_k) \\ \\ \dfrac{f_k(\overline{u}_k) - f_k(\bar{l}_k)}{\overline{u}_k - \bar{l}_k} & \text{if } f_k(\overline{u}_k) \ne f_k(\bar{l}_k) \end{cases} \qquad k = 1, 2, ..., n$$

and

$$s_k = \begin{cases} f_k(\bar{l}_k) & \text{if } f_k(\overline{u}_k) = f_k(\bar{l}_k) \\ \\ f_k(\bar{l}_k) - \dfrac{f_k(\overline{u}_k) - f_k(\bar{l}_k)}{\overline{u}_k - \bar{l}_k} \times \bar{l}_k & \text{if } f_k(\overline{u}_k) \ne f_k(\bar{l}_k) \end{cases} \qquad k = 1, 2, ..., n$$

Now consider the following optimization problem

$$\overline{\textbf{NLP}}\textbf{(node):} \quad \text{Minimize} \quad h(x) = \sum_{k=1}^{n} h_k(x_k)$$

$$\text{subject to}$$

$$\sum_{k=1}^{n} g_k(x_k) \leq g$$

$$\bar{l}_k \leq x_k \leq \bar{u}_k \,,\; k = 1, 2, ..., n$$

This nonlinear program is convex and has exactly the same feasible set as program NLP. Let $x^* = (x_k^*)_{k=1,2,...,n} \in \mathrm{K} \subseteq \mathbb{R}^n$ be an optimal solution of NLP, $f^* = f(x^*)$, and $x^{**} = (x_k^{**})_{k=1,2,...,n} \in \mathrm{K} \subseteq \mathbb{R}^n$ be an optimal solution of $\overline{\text{NLP}}$(node). Then the following result holds between the optimal values of those programs.

Theorem 3. $h(x^{**}) \leq f^* \leq f(x^{**})$.

Since $\overline{\text{NLP}}$(node) is a convex program, then its optimal value can be obtained as a stationary point (Karush–Kuhn–Tucker) of f on K (Bazaraa et al., 1993), which can be found by using a local nonlinear optimization algorithm (Bazaraa et al., 1993; Nocedal and Wright, 1999), such as `MINOS` (Murtagh and Saunders, 1987).

Theorem 3 also implies that $f(x^{**})$ is an upper bound for the optimal value f^* of the program NLP. Therefore, an upper bound associated with each node of the tree may also be computed using the optimal solution of the convex program $\overline{\text{NLP}}$(node), which is used to find a lower bound in a current node. The optimal solution x^{**} of the corresponding Convex Program $\overline{\text{NLP}}$(node) is a feasible solution for NLP(node), and can therefore be used as an initial point for the application of a local optimization algorithm. This may find a better upper bound by computing a stationary point for the function f in that feasible set. Since finding a stationary point is time consuming and usually does not lead into an improvement of the current upper bound, in practice the local optimization algorithm is only applied at the root node or when $f(x^{**})$ is quite close to the best upper bound found so far.

The implementation of the branch-and-bound algorithm requires a criterion for the choice of the node from all the open nodes in a current iteration. Another important point is related with the branching of the tree from each current node, that is, with the choice of the variable for branching.

As discussed before, each node of the tree has an associated lower bound that is smaller than the current upper bound. Whenever an upper bound is updated, all nodes with a lower bound equal to or greater than the new upper bound are discarded. If there are still some open nodes in the tree, then the chosen node is associated with the higher lower bound. As reported in Baptista (2004), this procedure is very easy to implement and works well in practice.

The variable for branching the tree from the node previously chosen is chosen using an adaptation of the heuristic rule presented in Konno and Wijayanayake (2001). Let $x^{**} = (x_k^{**})_{k=1,2,\ldots,n} \in \mathbb{R}^n$ be an optimal solution of the program $\overline{\text{NLP}}$(node), associated with the current node. Then it is chosen the variable x_k whose value in this optimal solution, x_k^{**}, maximizes $f_k(x_k) - h_k(x_k)$, for $k = 1, 2, ..., n$. So if j is the largest index corresponding to

$$x_j^{**} = \mathit{arg\ max} \; \{ \, f_k(x_k^{**}) - h_k(x_k^{**}) \;|\; k = 1, 2, ..., n \, \} \tag{17}$$

then x_j is the chosen variable.

After these considerations, the steps of the branch-and-bound algorithm can be described.

Branch-and-Bound Algorithm

Step 0: Initialization – Let $L = \{0\}$ be the initial list of open nodes and $\varepsilon > 0$ a tolerance. Compute the optimal solution x^{**} of the convex nonlinear program $\overline{\text{NLP}}(0)$ and the lower bound associated with this node $\text{LB}(0) = h(x^{**})$. Find an upper bound UB by computing a stationary point x^* of f in the set K of NLP. If $\dfrac{\text{UB} - \text{LB}(0)}{\max\{\text{UB}, 1\}} < \varepsilon$, then x^* is the global minimum of f in K and stop.

Step 1: Selection of the Node – If $L = \emptyset$, terminate the algorithm, with a global minimum x^* corresponding to the value of UB, that is, satisfying $f(x^*) = UB$. Otherwise, choose a node $t \in L$ corresponding to the largest value of $\text{LB}(k)$, for all $k \in L$. Set $L = L - \{t\}$. If $\dfrac{\text{UB} - \text{LB}(t)}{\max\{\text{UB}, 1\}} < \varepsilon$, repeat **Step 1**. Otherwise, go to **Step 2** with the optimal solution x^{**} of the corresponding $\overline{\text{NLP}}(t)$.

Step 2: Computation of an Upper Bound – If $f(x^{**}) < \text{UB} + \delta$, for a given $\delta \geq 0$, find a stationary point x^* of f in the feasible set of $\text{NLP}(t)$ by using a Local Algorithm with initial point x^{**} and set

$$\text{UB} = \min\{\text{UB},\; f(x^*)\}$$

Step 3: Branching and Computation of Lower Bounds – Let $|L|$ be the number of elements of L and x^{**} be the optimal solution of the $\overline{\text{NLP}}(t)$. Find the index j associated with criterion (17). Add two nodes $|L| + 1$ and $|L| + 2$ to the list L, with corresponding nonlinear programs

NLP$(|L| + 1)$**:** $\text{NLP}(t)$ with the replacement of constraint $\overline{l}_j \leq x_j \leq \overline{u}_j$ by

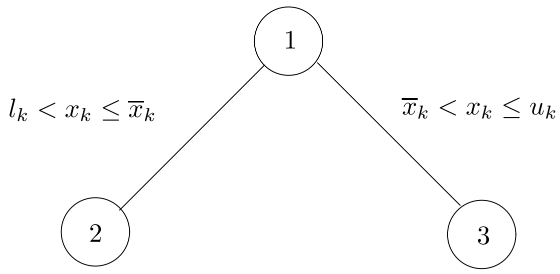
$$\overline{l}_j \leq x_j \leq \frac{\overline{l}_j + \overline{u}_j}{2}$$

NLP$(|L| + 2)$**:** $\text{NLP}(t)$ with the replacement of constraint $\overline{l}_j \leq x_j \leq \overline{u}_j$ by

$$\frac{\overline{l}_j + \overline{u}_j}{2} \leq x_j \leq \overline{u}_j$$

Compute lower bounds LB($|L|+1$) and LB($|L|+2$) by solving the corresponding programs $\overline{\text{NLP}}(|L|+1)$ and $\overline{\text{NLP}}(|L|+2)$. If $\overline{\text{NLP}}(|L|+1)$ (or $\overline{\text{NLP}}(|L|+2)$) is not feasible remove that node from the list L. Go to **Step 1**.

A branch-and-bound algorithm was described, which can process the optimization problem NLP (11) – (13) by assuming that, for each $k = 1, 2, ..., n$, the function f_k is concave or convex in the initial interval $[\, l_k\, ,\, u_k\,]$. If this is not the case, then, by theorem 2, there is $\overline{x}_k \in [\, l_k\, ,\, u_k\,]$ such that f_k is strictly convex on $[\, l_k\, ,\, \overline{x}_k\,]$ and strictly concave on $[\, \overline{x}_k\, ,\, u_k\,]$. Two daughter nodes can then be generated from the root such that



Then f_k is strictly convex on $[\, l_k\, ,\, \overline{x}_k\,]$ and strictly concave on $[\, \overline{x}_k\, ,\, u_k\,]$ and the branch-and-bound algorithm can be applied from now on. Therefore, the algorithm should incorporate in step 0 (initialization) a preprocessing phase, where 2^p nodes are generated from the root, with p the number of functions f_k whose second derivative vanishes into the interior of the interval $[\, l_k\, ,\, u_k\,]$. By using this procedure, the functions f_k are either strictly convex or strictly concave on their corresponding intervals at all the open nodes of the list and the branch-and-bound algorithm can be applied without any modification.

It should be added that the number p of functions f_k such that $f_k''(\overline{x}_k) = 0$ for $l_k < \overline{x}_k < u_k$ is smaller than or equal to the number of pipes with installed valves. As discussed in Section 6, this number is in practice about 10% of the total number of pipes. Furthermore, most of these functions f_k are concave or convex on their corresponding intervals $[\, l_k\, ,\, u_k\,]$. For instance, for the case-study reported in Section 6, the total number of pipes is 21, the number of pipes with installed valves is 2 and only one function f_k is neither strictly convex nor strictly concave on the whole interval $[\, l_k\, ,\, u_k\,]$. This means that $p = 1$ and there are two nodes in the list L at the beginning of the branch-and-bound procedure.

5 Computation of a Feasible Solution for the Model with Standard Values

The original objective of the hydraulics problem dealt with in this paper is the determination of values for the decision variables from a range of previously known fixed commercial values. In fact, each variable x_k represents the interior diameter of the pipe k of the water distribution pipe circuit ($k = 1, 2, ..., n$). So each x_k must assume a value from a discrete range of standard values in order to minimize the installation global cost, that is, x_k has to satisfy

$$x_k \in \{ m_1, m_2, ... m_l \} \quad , \forall\, k = 1, 2, ..., n$$

with fixed m_i, $i = 1, 2, ..., l$ and are written in ascending order. In order to describe how to find a feasible solution for the original Nonlinear Program with Standard Values (NLPSV), let $x^* = (x_k^*)_{k=1,2,...,n} \in \mathbb{R}^n$ be the optimal solution of the program NLP, obtained by the branch-and-bound algorithm. For all $k = 1, 2, ..., n$, the functions

$$g_k : x_k \longmapsto g_k(x_k) = d_k\, x_k^{-4.75}\, (\, e_k + q_k\, x_k \,)$$

are strictly decreasing in $[\, l_k \,,\, u_k \,]$. As the constraint

$$\sum_{k=1}^{n} g_k(x_k) \leq g \tag{18}$$

is an inequality $\leq$, rounding to the standard value $\overline{x}_k$ immediately above to x_k^* does not destroy the feasibility of the solutions. So a feasible solution for the program NLPSV may be computed by applying this updating procedure n times for each variable x_k.

Due to the monotonicity of the functions g_k, this algorithm always finds a feasible solution to the NLPSV. It may possible to obtain a solution for NLPSV with a smaller value for the objective function by simply updating each variable x_k^* of the optimal solution of NLP program to the standard value $\tilde{x}_k$ immediately below to x_k^* whenever $f_k(\tilde{x}_k) < f_k(\overline{x}_k)$. The updated solution $y = (y_k)_{k=1,2,...,n} \in \mathbb{R}^n$ is then defined by

$$y_k = \begin{cases} \overline{x}_k & \text{if } f_k(\overline{x}_k) \leq f_k(\tilde{x}_k) \\ \tilde{x}_k & \text{otherwise} \end{cases}$$

If this updated solution $y = (y_k)$ is feasible to NLPSV, the process terminates. Otherwise, let

$$K = \{\, k \,:\, y_k < x_k^* \,\}$$

For each $k \in K$, let r be the index defined by

$$x_r^* - y_r = \max \{\, x_k^* - y_k \,:\, k \in K \,\}$$

Then the solution $y = (y_k)$ is updated by

$$y_k = \begin{cases} y_k & \text{if } k \neq r \\ \overline{x}_r & \text{if } k = r \end{cases}$$

where $\overline{x}_r$ is the standard value immediately above to x_r^*. If this solution is feasible for the NLPSV then the procedure terminates. Otherwise update K by $K = K - \{r\}$ and repeat the procedure until a feasible solution of NLPSV is at hand. It is obvious that a maximum of $|K|$ steps are required to find a feasible solution to the NLPSV, where $|K|$ is the number of elements of the initial set K.

The lower and upper bounds for the optimization problem to be solved by the branch-and-bound algorithm should be standard values. A simple choice for these bounds can be achieved as shown below.

Let $l_k, u_k,\ k = 1, 2, ..., n$ be the lower and upper bounds of the nonconvex program considered in Section 2 and $\{ m_1, m_2, ... m_l \}$ be the set of standard values for the diameters. Then for each $k = 1, 2, ..., n$, set

$$l_k = m_{r_k} \text{ and } u_k = m_{s_k}$$

where

$$m_{r_k} - l_k = \min \{ m_i - l_k > 0,\ i = 1, 2, ..., l \}$$

$$u_k - m_{s_k} = \min \{ u_k - m_j > 0,\ j = 1, 2, ..., l \}$$

It is important to add that these lower and upper bounds should be constructed before applying the branch-and-bound algorithm to process the NLP optimization problem.

6 Computational Experience

The experiments reported in this section were performed using a PC with 1.83 GHz Intel Core Duo T2400 processor and 1024 Mb RAM memory, running Windows XP Home Edition. The branch-and-bound algorithm was implemented within the `GAMS` environment and the active–set code `MINOS` (Murtagh and Saunders, 1987) of `GAMS` collection has been used for processing both the nonlinear programs $\overline{\text{NLP}}$(node) and NLP(node), which compute lower and upper bounds associated with each node respectively. Running times presented in this section are given in CPU seconds, excluding *inputs* and *outputs*. The results reported in this section relate to a case-study reported in Silva-Afonso (2001), which is discussed below.

(i) A Case-study

Consider the six-storey tower of a hospital building. Apart from the top one, all the floors are built in two symmetrical rows, with a central pantry (CP) equipped with a sink (Ll). Eight private rooms (QP) with a bathroom are planned for each row. All the rooms have a bidet (Bd), a toilet with flush cistern (Br), a wash basin (Lv) and a shower (Ch). Also planned for each row are:

- a nursing service room (TE), with a wash basin (Lv);
- a bed disinfection room (DC), with a bed disinfection device (Mdc);
- a room for soiled materials (SJ), with a hospital pan washer (Pd), a bedpan washing machine (Mla) and a wash basin (Lv);

- a treatment room (TR), with a wash basin (Lv) and a service sink (Bl).

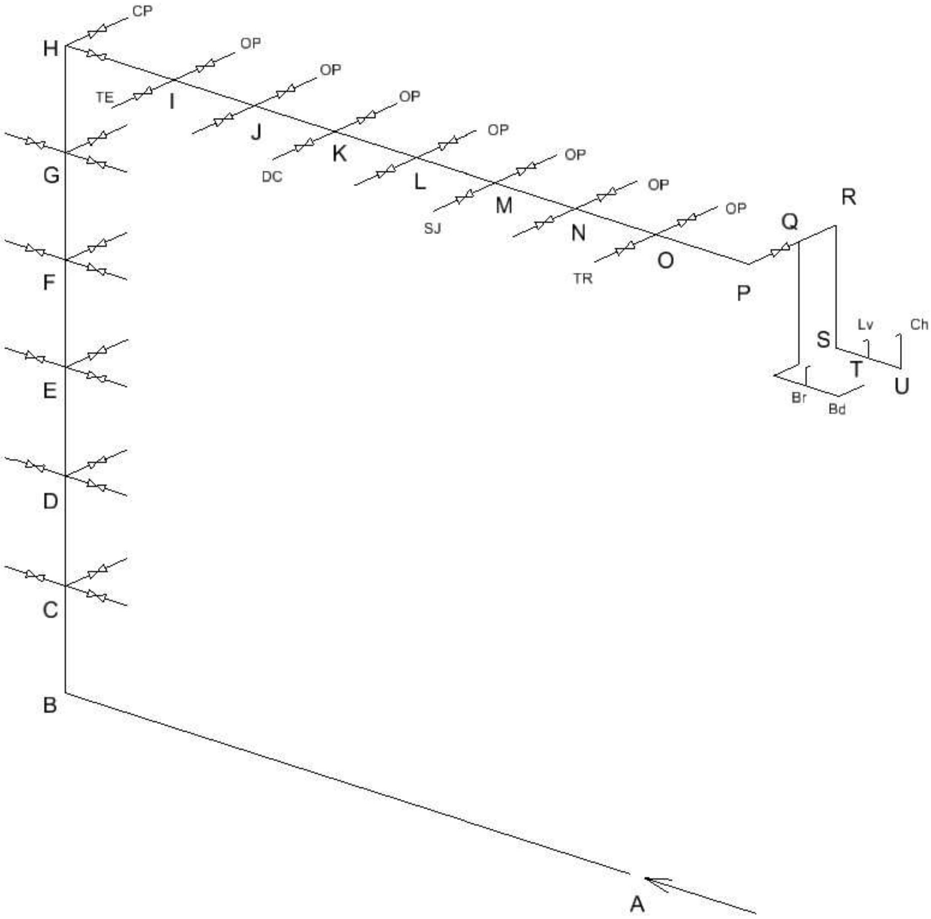


Figure 2: Axonometric Perspective.

Figure 2 shows the axonometric perspective of the circuit considered the least favourable, corresponding to the shower (Ch) supply in the last bathroom on the 6th floor. The pipes are made of stainless steel. The minimum pressure on the entry of the circuit (A point) is of $500\,kPa$ and, for comfort reasons and equipment requirements, minimum residual pressures of $250\,kPa$ should be guaranteed. The valves to be installed (in pipes H-I and P-Q) are globe valves, which have high head losses, even in the outlet position. Their respective values have therefore been considered in the computation and have been fixed from tables found in the literature. The design flows have been calculated on the basis of the Portuguese regulations. Design pipe lengths have been computed on the basis of the increase factor 1.25 (which thus corresponds to the percentage of increase $\rho = 25\%$), applied to the real pipe lengths. Besides the velocity bounds due to regulations, given by (4) and (5), the bound (6) proposed in Silva-Afonso (2001) has also been assumed inside the pipes, for a high comfort level.

(ii) Computational Results with the Model

Table 1, from Silva-Afonso et al. (2002), illustrates the results from applying the traditional water piping system sizing criteria to the example described in this section. The maximum permitted velocity criterion, which leads to the least cost solution, does not provide an adequate solution in the present case, due to insufficient available head, establishing that the residual pressure in the least favourable fixture ($237.1\,kPa$) is lower than the minimum value required ($250\,kPa$). As mentioned in Section 1, the usual procedure for this situation consists of a new sizing on the basis of the maximum total permitted head loss. This is achieved by computing a mean value for the unit friction loss in the circuit, which allows the computation of the diameters for the various pipes, such that the solution is adequate for its purpose. In this example all boundary conditions are now satisfied but the overall cost of the circuit rises from 2610.00 to 2895.00 euros, which represents an increase of about 11%.

Sizing criterion	Residual pressure in Ch (kPa)	Circuit cost (euros)
Maximum permitted velocity criterion	237.1 ($<$ 250)	2610.00
Maximum permitted total head loss criterion	256.4	2895.00

Table 1: Sizing of the critical circuit using traditional criteria.

In order to apply the economic design criterion to this situation using the formulation proposed in Section 2, it is necessary to know the constants L_k, a_k, b_k, c_k, α_k, β_k, γ_k, η_k, g (data of the problem) and d_k, e_k, f_k (computed from those data, according to the expressions introduced in Section 2), $k = 1, 2, ..., n$, presented in the model. According to Figure 2, assuming that the pipes in the circuit are sequentially numbered from A-B to U-Ch, there are 21 pipes. Then $n = 21$ and $T = \{1, 2, \ldots, 21\}$. As valves only have to be installed in pipes H-I (corresponding to $k = 8$) and P-Q ($k = 16$), the local head losses associated with these valves cannot to be discarded and

$$V = S = \{8, 16\} \Rightarrow v_8 = v_{16} = 1 \;\; (v_k = 0, \text{ for } k \in T \setminus \{8, 16\})$$

Problem data are as follows:

$$C = 580; \; \rho = 25; \; g\,(= \Delta H_M) = 4.60$$

and for each pipe $k = 1, 2, ..., 21$,

$$a_k = 3.2 \times 10^3; \; b_k = 873; \; c_k = 4.5$$

$\alpha_k = 450 \times 10^3$; $\beta_k = 80.5 \times 10^3$; $\gamma_k = 2.2 \times 10^3$; $\eta_k = 21.3$

The lengths L_k and design flows Q_k are presented in the columns 3 and 4 of Table 2, respectively. Lower and upper bounds l_k and u_k, $k = 1, 2, ..., 21$, are first computed by

$$l_k = max\left\{0.921\,Q_k^{0.5}\,,\, 0.400\,Q_k^{0.37}\right\},\ \text{ for } k \in T_1$$

$$l_k = max\left\{0.798\,Q_k^{0.5}\,,\, 0.400\,Q_k^{0.37}\right\},\ \text{ for } k \in T_2$$

and

$$u_k = 1.596\,Q_k^{0.5},\ \text{ for } k \in T$$

where $T_1 = \{2, \ldots, 21\}$ and $T_2 = \{1\}$ (T_1 and T_2 constitute a partition of the pipe circuit set, T) (Silva-Afonso, 2001). Their values of l_k and u_k, $k = 1, 2, ..., 21$, are displayed in Table 2.

The following set of standard sizes is considered and correspond to the range of commercially available diameters (Silva-Afonso, 2001) (expressed in meters):

$$\{0.0138,\ 0.0166\,,0.0206\,,0.0264\,,0.0330\,,0.0396\,,0.0516\,,0.0603\,,0.0721\,,0.0849\,,0.104\}$$

As discussed earlier, the lower and upper bounds of the program NLP should be standard values. By using the sizes in Table 2 and these standard commercial diameters, the lower and upper bounds for all the variables can be computed according to the process explained in Section 5.

It is easy to see that the functions f_k but f_8 are either strictly concave or strictly convex on their corresponding intervals $[\,l_k\,,\,u_k\,]$. Therefore, the algorithm generates two nodes in the initialization step, according to the procedure described in Section 4.

Table 3 gives the results for the solution of the nonconvex program NLP by the branch-and-bound algorithm $\left(\varepsilon = 10^{-6},\ \delta = 0\right)$. Since the global minimum of this program is not feasible for the initial problem (whose interior pipe diameters must belong to the range of standard sizes), the procedure for finding a solution with standard sizes discussed in Section 5 was employed. The solution found is presented in Table 4 and corresponds to a percentage saving of about 6.2% compared with the best solution computed using the traditional sizing criteria, which is, according to Table 1, the maximum total permitted head loss. Furthermore, the need to adjust the diameters to the range of commercially available diameters implied an increase of about 0.44% in the cost of the solution (rising from 2714.177 to 2726.080 euros).

The branch-and-bound algorithm performed quite well for finding a global optimal solution for the nonconvex program NLP. In fact, the number of iterations (that is, the number of times that the branching process is used) is small for a problem with 21 variables. Furthermore, the execution time is clearly quite acceptable for the computation of a global minimum for a nonconvex program.

Pipes		Lengths	Design	l_k	u_k
k	designation	L_k (m)	flows Q_k (m^3/s)		
1	A-B	30.00	0.00441	0.054	0.106
2	B-C	3.70	0.00441	0.061	0.106
3	C-D	3.70	0.00397	0.058	0.101
4	D-E	3.70	0.00348	0.054	0.094
5	E-F	3.70	0.00292	0.050	0.086
6	F-G	3.70	0.00224	0.044	0.076
7	G-H	3.70	0.00126	0.034	0.057
8	H-I	3.50	0.00124	0.034	0.056
9	I-J	4.00	0.00116	0.033	0.054
10	J-K	4.00	0.00109	0.032	0.053
11	K-L	4.00	0.00099	0.031	0.050
12	L-M	4.00	0.00091	0.030	0.048
13	M-N	4.00	0.00071	0.027	0.043
14	N-O	4.00	0.00059	0.026	0.039
15	O-P	4.00	0.00038	0.022	0.031
16	P-Q	0.50	0.00038	0.022	0.031
17	Q-R	1.50	0.00038	0.022	0.031
18	R-S	3.00	0.00025	0.019	0.025
19	S-T	0.60	0.00025	0.019	0.025
20	T-U	1.00	0.00015	0.015	0.020
21	U-Ch	0.95	0.00015	0.015	0.020

Table 2: Problem Data.

Time of execution (seconds)	Number of nodes	Cost of the solution (euros)	
		optimal solution	solution with standard sizes
11.00	321	2714.177	2726.080

Table 3: Optimal Solution of the Nonlinear Program and Feasible Solution of the Problem with Standard Sizes.

Even though this case-study is concerned with a relatively small critical circuit, it obtained a notable real saving in percentage terms of nearly 6%, through the use of the economic design criterion and the proposed formulation. This new optimization model could well have considerable interest for large scale systems with very extensive critical circuits, characteristics

Pipe	Diameters (meters)	
k	optimal solution	commercial range
1	0.0604	0.0603
2	0.0721	0.0721
3	0.0603	0.0603
4	0.0603	0.0603
5	0.0524	0.0516
6	0.0516	0.0516
7	0.0396	0.0396
8	0.0475	0.0516
9	0.0385	0.0396
10	0.0377	0.0396
11	0.0366	0.0330
12	0.0356	0.0330
13	0.0330	0.0330
14	0.0309	0.0330
15	0.0264	0.0264
16	0.0264	0.0264
17	0.0264	0.0264
18	0.0206	0.0206
19	0.0206	0.0206
20	0.0166	0.0166
21	0.0166	0.0166
Cost (euros)	2714.177	2726.080

Table 4: Solutions for the Economic Design Criterion

that are found in specialized buildings such as hospitals, hotels, shopping centers and airports.

7 Conclusion

This paper describes the formulation and solution of an engineering optimization problem in hydraulics which is related to the application of a new economic design criterion for a water distribution piping system. This problem is of particular interest for networks with extensive critical circuits, involving inadequate residual pressures or pressure fluctuations, normally associated with specialized buildings, such as hospitals, hotels, shopping centers and airports.

This criterion leads into a nonconvex nonlinear program with some standard values that have to be met. A branch-and-bound algorithm has been proposed to find a global minimum for the nonconvex program. A simple procedure to find a feasible solution satisfying these standard values has also been introduced.

The model has proved quite practical in a real-life application. The authors believe that the proposed model and methodology will be very useful in the solution of more complex hydraulics problems. This will certainly be a future topic of their research interests.

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