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On the optimal design of river fishways

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Abstract A river fishway is a hydraulic structure enabling fish to overcome stream obstructions such as dams in hydroelectric power plants. The main aim of this paper is to present an application of mathematical modeling and optimal control theory to an ecological engineering problem related to preserve and enhance natural stocks of fish migrating between saltwater and freshwater. Particularly, we improve the optimal shape design of a fishway. This problem is formulated within the framework of the optimal control of partial differential equations, approximated by a discrete optimization problem, and solved by using both a gradient method (a Spectral Projected-Gradient algorithm) and a derivative-free method (the Nelder-Mead algorithm). Finally, numerical results are compared and analyzed for a standard real-world situation.

Keywords Fishway · Shape design · Numerical optimization · Optimal control · Partial differential equations

1 Introduction

Many species of fish attempt migrations on a regular basis, on time scales varying from daily to annual, and with distances ranging from a few meters to thousands of

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kilometers. In this work, we pay attention to diadromous fish which migrate between fresh and salt water. The best known diadromous fish are salmon (*salmo salar*), trout (*salmo trutta*), eel (*anguilla anguilla*), sturgeon (*acipenser sturio*), lamprey (*lampetra fluviatilis*, *petromyzon marinus*), barbel (*barbus bocagei*), carp (*cyprimus carpio*), and perch (*perca fluviatilis*). There are three main types of diadromous fish, depending on their specific migration patterns: anadromous, catadromous and amphidromous. Anadromous fish spend most of their adult lives in saltwater, and migrate to freshwater rivers and lakes to reproduce. Anadromous fish species include lamprey, sturgeon, salmon, and trout. More than half of all diadromous fish in the world are anadromous. Catadromous fish spend most of their adult lives in freshwater, and migrate to saltwater to spawn. Juvenile fish migrate back upstream where they stay until maturing into adults, at which time the cycle starts again. One of the most common catadromous species is the eel. About one quarter of all diadromous fish are catadromous. Finally, amphidromous species move between estuaries and coastal rivers and streams, usually associated with the search for food or refuge rather than reproduction need. Amphidromous fish can spawn in either freshwater or in a marine environment. Less than one fifth of all diadromous fish are amphidromous, for instance, the bull shark (*carcharhinus leucas*).

Fish usually migrate because of reproductive needs. As a widely known example, salmon are able to swim hundreds of kilometers upriver. The salmon - that will be the type of fish considered in our study - hatch in small freshwater streams. From there they migrate to sea to mature, where they live for three to six years. When mature, the salmons return to the same streams where they were hatched to spawn. When an artificial barrier is constructed in a stream (for example, a weir or a dam in a hydroelectric power plant) European legal regulations force the installation of a fishway in order to allow the salmon to overcome the barrier.

Fishways are hydraulic structures placed on (or around) man-made barriers to assist the natural migration of diadromous fish. An exhaustive overview on the design and management of river fishways (also known as fish ladders, fish passes or fish steps) can be found in the interesting monograph of Clay [10]. In the literature, apart from some nature-like structures as the rock-ramp fishways, three main types of fishways are studied: the pool and weir type [28], the Denil type [25], and the vertical slot type [27].

Pool and weir fishways were the earliest type constructed (first recorded attempts to construct this type of fishway were made in Europe in the 17th century), and are still built with the addition of orifices in their walls. A pool and weir fishway consists of a number of pools formed by a series of weirs. The fish passes over a weir by swimming fast enough to jump over the weir. After a short rest in the pool, the fish then passes over the next weir, and so on, until the ascent is complete. The success of this type of fishway depends on the maintenance of water levels, which can be facilitated by the provision of a set of orifices in the weir walls close to the floor.

The Denil fishway is essentially a straight rectangular flume provided with closely spaced baffles or vanes on the bottom and sides. The first of the classical works of G. Denil on the scientific design of fish-passes was already published in 1909 in Belgium. Of the many types of Denil fishways studied in the scientific literature, the

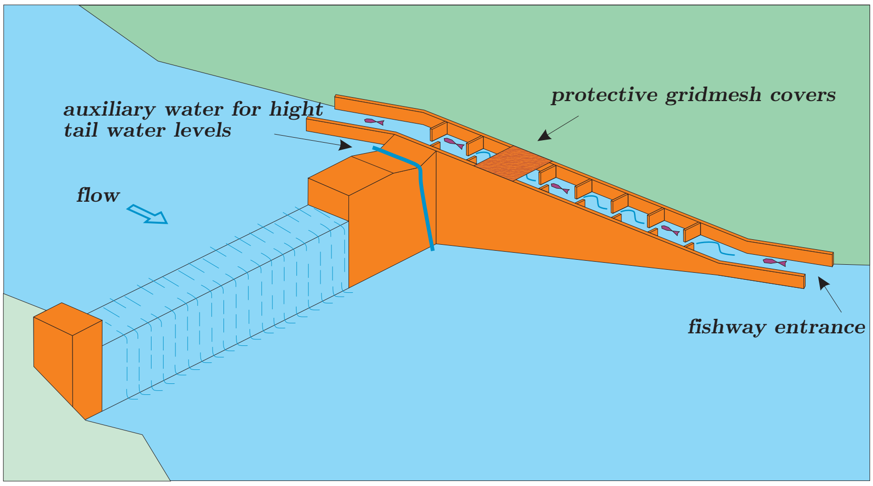

Fig. 1 Schematic drawing of a vertical slot fishway

more commonly used are the standard Denil fishway, the Larinier fishway, and the more sophisticated Alaskan steeppass fishway [26].

Nevertheless, here we deal with a third type of fishway, which is the more generally adopted for upstream passage of fish in stream obstructions: the vertical slot fishway. This fishway consists of a rectangular channel with a sloping floor that is divided into a reduced number of pools (see Fig. 1). Water runs downstream in this channel, through a series of vertical slots from one pool to the next one below. The water flow forms a jet at the slot, and the energy is dissipated by mixing in the pool. The fish ascends, using its burst speed, to get past the slot, then it rests in the pool until the next slot is tried [8].

During the last few decades, much attention has been paid, both from theoretical and experimental viewpoints, to the hydraulic characteristics, the flow regimes, and the turbulence structures in all types of river fishways (as can be seen, for instance, in the pioneering works of Rajaratnam *et al.* [29,30,33,11,12,18]). However, the fundamental role of a correct design in the fishway has received little attention. The following list covers some of the research done in this area: the work of Kim [16] (for the case of pool and weir fishways), the works of Odeh [23] and of Mallen-Cooper and Stuart [19] (for the design of Denil fishways), and - in a more general approach within the field of ecological/environmental engineering - the works of Karisch and Power [13], Weber and Joy [32], Yasuda *et al.* [34] and Richmond *et al.* [31], among others. However, the optimal design of a vertical slot fishway has only been previously analyzed by the authors [3–5] in a simple case.

As stated above, the objective of a fishway is enabling fish to overcome obstructions. In order to achieve this goal, water velocity in the fishway must be controlled. Specifically, this means that in the zone of the channel near the slots, the velocity of the water must be close to a desired velocity to allow the fish leap and swim up the fishway. In the rest of the fishway, the velocity must be close to zero to allow the fish rest. Moreover, in the entire channel, flow turbulence must be minimized. It is worth-

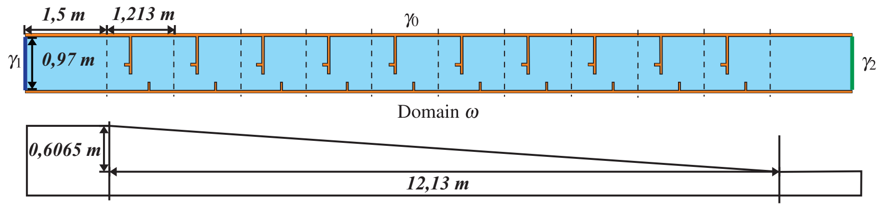

Fig. 2 Ground plant (domain ω with boundaries γ_0, γ_1, γ_2) and elevation of the fishway

while recalling here that the velocity of water in the channel has to be large enough to attract the fish to the fishway, but it cannot be so large that it washes fish back downstream or exhausts them to the point of inability to continue their journey upriver. Unfortunately, it is not difficult to find several examples of rivers where a wrong design of fishways has prevented the fish from gaining access to their spawning grounds and contributing to a catastrophic decline in their numbers.

Water velocity can be directly controlled by determining the optimal shape of the fishway, that is, the location and length of the baffles separating the pools. In this work, we are going to use mathematical modelling and optimal control theory to address the optimal design of a fishway. In order to do this, we begin by presenting a mathematical model (shallow water equations) to simulate the water velocity in a fishway and give a mathematical expression to evaluate the quality of that velocity field in terms of the fish capabilities. Next, we study the problem of the optimal design of a fishway: we describe the problem, formulate it as a shape optimization problem, and show that it can be approximated by a discrete optimization problem. A derivative-free method (the Nelder-Mead algorithm) and a gradient one (the Spectral Projected-Gradient algorithm) are proposed in last sections to solve this optimization problem. Finally, numerical results for a standard fishway are presented.

2 Numerical simulation of fishway hydrodynamics

Let $\omega \subset \mathbb{R}^2$ be the ground plant of a fishway consisting of a rectangular channel divided into a small number of pools with baffles and sloping floor, and two transition pools (one at the beginning and another one at the end of the channel) with no baffles and flat floor. A scheme of the standard fishway used in this paper can be seen in Fig. 2: water enters by the left side and runs downstream to the right side, while fish ascend in the opposite direction [4]. The number of pools (ten) and the dimensions of the full channel correspond to an experimental scale fishway reported by Puertas, Pena and Teijeiro [24]. The main distinction between previous contributions by the authors [3–5] and the present paper consists of the inclusion of a third extra baffle (which appears in the most usual vertical slot fishways) in order to control the flow in a more accurate way. Although the formulations of both problems are very similar, the number of parameters of the optimization problem increases from four to six. This makes the previously used direct search method (the Nelder-Mead algorithm) an excessively expensive method from a computational viewpoint, compelling us to the search for a more computationally efficient type of optimization algorithm.

Water flow inside the domain ω along the time interval $(0,T)$ is governed by the shallow water equations [3]:

$$\begin{cases} \dfrac{\partial H}{\partial t} + \nabla\cdot\mathbf{Q} = 0 & \text{in } \omega\times(0,T), \\ \dfrac{\partial \mathbf{Q}}{\partial t} + \nabla\cdot(\dfrac{\mathbf{Q}}{H}\otimes\mathbf{Q}) + gH\nabla(H-\eta) = \mathbf{f} & \text{in } \omega\times(0,T), \end{cases} \tag{1}$$

where

- $H(x,y,t)$ is the height of water at point $(x,y)\in\omega$ and at time $t\in(0,T)$,
- $\mathbf{u}(x,y,t)=(u,v)$ is the depth-averaged horizontal velocity of water,
- $\mathbf{Q}(x,y,t)=\mathbf{u}H$ is the areal flow per unit depth,
- g is the gravity acceleration,
- $\eta(x,y)$ represents the bottom geometry of the fishway,
- second member $\mathbf{f}$ collects all the effects of bottom friction, atmospheric pressure and so on.

These equations must be completed with a set of initial and boundary conditions. In order to do that, we need to define three different parts in the boundary of ω: the lateral boundary of the channel denoted by γ_0, the inflow boundary denoted by γ_1, and the outflow boundary denoted by γ_2. We also consider $\mathbf{n}$ the unit outer normal vector to boundary. Thus, we assume the normal flux and the vorticity to be null on the lateral walls of the fishway, we impose an inflow flux in the normal direction, and we fix the height of water on the outflow boundary, that is,

$$\begin{cases} H(0)=H_0, \quad \mathbf{Q}(0)=\mathbf{Q}_0 & \text{in } \omega, \\ \mathbf{Q}\cdot\mathbf{n}=0, \quad curl(\dfrac{\mathbf{Q}}{H})=0 & \text{on } \gamma_0\times(0,T), \\ \mathbf{Q}=q_1\,\mathbf{n} & \text{on } \gamma_1\times(0,T), \\ H=H_2 & \text{on } \gamma_2\times(0,T). \end{cases} \tag{2}$$

The use of the 2D shallow water equations (instead of the more expensive 3D equations) has been recently validated in [9], where a comparison of the 2D numerical results with velocity measurements (performed by using particle imaging velocimetry and acoustic Doppler velocimetry) allowed the validation of the numerical results.

Using the above notation we can give a mathematical expression to evaluate the water velocity in the fishway. We have to bear in mind two objectives:

(i) In the zone of the channel near the slots (say the lower third) the velocity must be as close as possible to a typical horizontal velocity c to allow the fish leap and swim up the fishway (c must be chosen bearing in mind that the maximum swimming speed for salmon is about $3\,ms^{-1}$), and in the remaining of the fishway the velocity must be close to zero so the fish can rest. In short, the velocity of water must be close to the following target velocity:

$$\mathbf{v}(x_1,x_2) = \begin{cases} (c,0), & \text{if } x_2 \leq \frac{1}{3}W, \\ (0,0), & \text{otherwise,} \end{cases} \tag{3}$$

where W is the width of the channel (in our case, as shown in Fig. 2, $W=0.97\,m$).

(ii) Flow turbulence must be minimized in all the channel in order to avoid fish disorientation, that is, the vorticity must be as reduced as possible.

According to this, if we fix a weight parameter $\xi \geq 0$ for the role of the vorticity and define the objective function

$$J = \frac{1}{2}\int_0^T \int_\omega \|\frac{\mathbf{Q}}{H} - \mathbf{v}\|^2 + \frac{\xi}{2}\int_0^T \int_\omega |curl(\frac{\mathbf{Q}}{H})|^2, \tag{4}$$

the water velocity $\mathbf{u} = \mathbf{Q}/H$ will be better for our purposes, when the value of the cost function J becomes smaller.

In order to evaluate J, we first need to solve the shallow water equations (1) with initial and boundary conditions (2). In this work we use an implicit discretization in time, upwinding the convective term by the method of characteristics, and Raviart-Thomas finite elements for the space discretization (the whole details of the numerical scheme can be seen in Bermúdez *et al.* [6]). For the time interval $(0,T)$, we choose a natural number N, consider the time step $\Delta t = T/N > 0$ and define the discrete times $t_k = k\Delta t$ for $k = 0,\ldots,N$. We also consider a Lagrange-Galerkin finite element triangulation τ_h of the domain ω. Thus, the numerical scheme provides, for each discrete time t_k, an approximated flux $\mathbf{Q}_h^k$ and an approximated height H_h^k, which are piecewise-linear polynomials and discontinuous piecewise-constant functions, respectively. This numerical procedure has shown a very good performance in several related problems previously analyzed by the authors, for instance, the optimal depuration levels in a wastewater treatment plant [20], the optimal location of wastewater outfalls [2], or the optimal purification of polluted waters [1]. Finally, with these approximated fields we can compute the approximated velocity $\mathbf{u}_h^k = \mathbf{Q}_h^k/H_h^k$, and approximate the value of J by

$$J_h^{\Delta t} = \frac{\Delta t}{2}\sum_{k=1}^{N}\sum_{E\in\tau_h}\{\int_E \|\mathbf{u}_h^k - \mathbf{v}\|^2 + \xi \int_E |curl(\mathbf{u}_h^k)|^2\}. \tag{5}$$

3 Optimal shape design of a fishway

In this section we study how to improve the optimal design of a fishway. As commented before, we can control the water velocity through the location and length of the baffles in the pools. To be consistent, for the channel described in the previous section, we assume that the structure of the ten pools with sloping floor has to be the same (the shape of the entire fishway is given by the shape of the first pool), and then we take the three midpoints corresponding to the end of the baffles in the first pool (points $a = (s_1,s_2)$, $b = (s_3,s_4)$ and $c = (s_5,s_6)$ in Fig. 3) as design variables.

We look for points a, b and c that provide the best velocity for the fish (i.e. minimizing the function J given by (4)). However, we must impose several design constraints: first, we assume that points a, b and c are inside the dashed rectangle of Fig. 3, that is, the following twelve geometrical relations must be satisfied (in order to avoid unnecessary symmetrical duplicate solutions):

$$\begin{cases} x_{min} = \frac{1}{4}1.213 \leq s_1,s_3,s_5 \leq \frac{3}{4}1.213 = x_{max}, \\ y_{min} = 0 \leq s_2,s_4,s_6 \leq \frac{1}{2}0.97 = y_{max}. \end{cases} \tag{6}$$

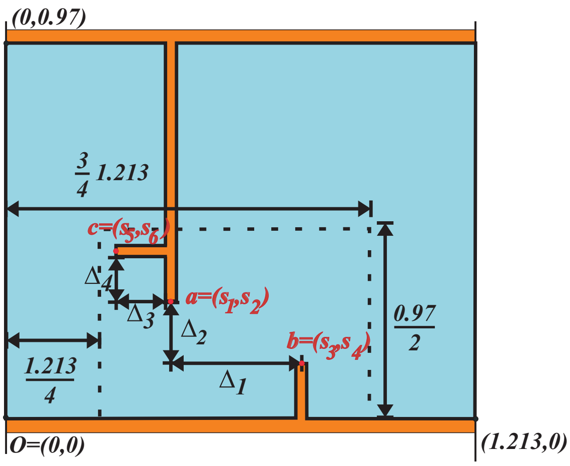

Fig. 3 Scheme of the first pool

The second type of constraints are related to the fact that the vertical slot must be large enough so that salmon can pass comfortably through it. This requirement translates into the two following linear constraints:

$$\begin{cases} \Delta_1 = s_3 - s_1 \geq 0.1 = h_1, \\ \Delta_2 = s_2 - s_4 \geq 0.05 = h_2. \end{cases} \tag{7}$$

If the half width of the baffle is $r = 0.0305\,m$, (standard datum as appeared in Puertas *et al.* [24]) we are actually imposing that the slot width must be, at least, of $\sqrt{(0.1-2r)^2 + 0.05^2} = 0.063\,m$.

Finally, the third type of constraints are related to structural stability, as given by the two additional linear constraints:

$$\begin{cases} \Delta_3 = s_1 - s_5 \geq \frac{1}{2} 0.0305 = d_1, \\ \Delta_4 = s_6 - s_2 \geq \frac{1}{2} 0.0305 = d_2. \end{cases} \tag{8}$$

Then, the optimization problem can be formulated as follows:

Problem $(\mathscr{P})$: Find the optimal shape of domain ω, specifically, find vector $s = (a,b,c) = (s_1,s_2,s_3,s_4,s_5,s_6)^T \in \mathbb{R}^6$ verifying constraints (6)-(8), in such a way that $\mathbf{Q}$ and H, given by the solution of the state system (1)-(2) on the fishway $\omega \equiv \omega(s)$, minimize the objective function $J \equiv J(s)$ defined by (4).

A mathematical analysis of a simpler related problem can be found in Alvarez-Vázquez *et al.* [3,4]. For its numerical resolution we propose two different types of methods: firstly, a derivative-free method (the Nelder-Mead algorithm), which is suitable for geometrical problems; secondly, a gradient method (the Spectral Projected-Gradient algorithm) where the necessary derivatives will be approached by finite difference approximations, and the projection computed *via* a linear complementarity problem. Both methods are described in the next sections. Finally, in the last section, the results achieved by both algorithms for a realistic case are analyzed and compared.

4 First approach: Derivative-free optimization

The Nelder-Mead (NM) simplex method [22] is a direct search method, which merely compares function values; the values of the objective function being taken from a set of sample points (simplex) are used to continue the sampling.

In order to minimize a given function $\Phi : \mathbb{R}^n \to \mathbb{R}$, the algorithm can be easily summarized in the following way: Recall that the convex hull of $n+1$ points in $\mathbb{R}^n$ not contained in the same n-hyperplane is called a n-simplex. The method constructs a sequence of simplices as approximations to a minimum point. The $n+1$ vertices $y_1, y_2, \ldots, y_{n+1}$ of each simplex are sorted according to the objective function values: $\Phi(y_1) \leq \Phi(y_2) \leq \ldots \leq \Phi(y_{n+1})$, and the worst vertex y_{n+1} is replaced with a new point $y(\nu) = (1+\nu)y - \nu\, y_{n+1}$, where y is the centroid of the convex hull of $\{y_1, y_2, \ldots, y_n\}$, that is, $y = (y_1 + \ldots + y_n)/n$. The value of ν is selected from a sequence $-1 < \nu_\delta < 0 < \nu_\gamma < \nu_\beta < \nu_\alpha$ (typical values are $\nu_\delta = -0.5$, $\nu_\gamma = 0.5$, $\nu_\beta = 1$, $\nu_\alpha = 2$) by rules given in the following algorithm:

While $\Phi(y_{n+1}) - \Phi(y_1)$ is not small enough, compute $y(\nu_\beta)$ and $\Phi_\beta = \Phi(y(\nu_\beta))$. Then:

(a) (Reflection) If $\Phi_\beta < \Phi(y_1)$, compute $\Phi_\alpha = \Phi(y(\nu_\alpha))$. If $\Phi_\alpha < \Phi_\beta$, replace y_{n+1} with $y(\nu_\alpha)$; otherwise replace y_{n+1} with $y(\nu_\beta)$. Go to (f).
(b) (Expansion) If $\Phi(y_1) \leq \Phi_\beta < \Phi(y_n)$, replace y_{n+1} with $y(\nu_\beta)$ and go to (f).
(c) (Outside contraction) If $\Phi(y_n) \leq \Phi_\beta < \Phi(y_{n+1})$, compute $\Phi_\gamma = \Phi(y(\nu_\gamma))$. If $\Phi_\gamma \leq \Phi_\beta$, replace y_{n+1} with $y(\nu_\gamma)$ and go to (f); otherwise go to (e).
(d) (Inside contraction) If $\Phi(y_{n+1}) \leq \Phi_\beta$, compute $\Phi_\delta = \Phi(y(\nu_\delta))$. If $\Phi_\delta < \Phi(y_{n+1})$, replace y_{n+1} with $y(\nu_\delta)$ and go to (f); otherwise go to (e).
(e) (Shrinking) For $k = 2, \ldots, n+1$, set $y_k = y_1 + (y_k - y_1)/2$.
(f) (Sorting) Resort the resulting vertices according to Φ values.

Although the NM algorithm is not guaranteed to converge in the general case, it has good convergence properties in low dimensions (see Lagarias *et al.* [17] for a detailed analysis of the convergence in one and two dimensions under convexity requirements). Moreover, in order to prevent stagnation at a non-optimal point, we use a modification proposed by Kelley [15]: we define the simplex gradient $D\Phi = V^{-T}\Delta\Phi$, where V and $\Delta\Phi$ are the matrices given by:

$$\begin{aligned} V &= (y_2 - y_1, y_3 - y_1, \ldots, y_{n+1} - y_1) \\ \Delta\Phi &= (\Phi(y_2) - \Phi(y_1), \Phi(y_3) - \Phi(y_1), \ldots, \Phi(y_{n+1}) - \Phi(y_1)) \end{aligned} \tag{9}$$

Thus, when stagnation is detected, we modify the simplex by an oriented restart, replacing it by the new smaller simplex $\hat{y}_1 = y_1$, $\hat{y}_j = \hat{y}_1 - \beta_{j-1} e_{j-1}$, $2 \leq j \leq n+1$, where e_k denotes the k-th vector of the canonical basis of $\mathbb{R}^n$, and

$$\beta_k = \begin{cases} \frac{\sigma}{2}, & \text{if } (D\Phi)_k \geq 0, \\ -\frac{\sigma}{2}, & \text{otherwise,} \end{cases} \tag{10}$$

for the typical length $\sigma = \min\limits_{2 \leq j \leq n+1} \|y_j - y_1\|$.

On the other hand, since the NM algorithm is specified for unconstrained problems, we need to reformulate our original problem ($\mathscr{P}$) by means of a penalty argument in order to deal with the sixteen linear constraints (6)-(8). Particularly, for a large enough parameter $\mu > 0$, we approximate $(\mathscr{P})$ by the unconstrained optimization problem:

$$\min_{s\in\mathbb{R}^6} \Phi(s) \tag{11}$$

where, for $s = (s_1, s_2, \dots, s_6) \in \mathbb{R}^6$, the value of $\Phi(s)$ can be computed from the following algorithm:
Step i. (Domain construction) Consider the corresponding domain $\omega(s)$, and an associated triangulation $\tau_h(s)$.
Step ii. (State system resolution) Solve the state system (1)-(2) on $\omega(s)$ as it was proposed in the previous section, and compute the value of $J(s)$ by expression (5).
Step iii. (Constraints) Define $\Psi(s)$ in such a way that $\Psi(s) \leq 0 \Leftrightarrow s$ verifies (6)-(8), i.e., consider, for instance,

$$\begin{aligned}\Psi(s) = \max\{&\tfrac{1}{4}\,1.213 - s_1, \tfrac{1}{4}\,1.213 - s_3, \tfrac{1}{4}\,1.213 - s_5,\\ &s_1 - \tfrac{3}{4}\,1.213, s_3 - \tfrac{3}{4}\,1.213, s_5 - \tfrac{3}{4}\,1.213, -s_2, -s_4, -s_6,\\ &s_2 - \tfrac{1}{2}\,0.97, s_4 - \tfrac{1}{2}\,0.97, s_6 - \tfrac{1}{2}\,0.97, 0.1 - s_3 + s_1,\\ &0.05 - s_2 + s_4, \tfrac{1}{2}\,0.0305 - s_1 + s_5, \tfrac{1}{2}\,0.0305 - s_2 + s_6\}.\end{aligned} \tag{12}$$

Step iv. (Penalty) Compute the value of the discrete penalty function

$$\Phi(s) = J(s) + \mu \max\{\Psi(s), 0\}. \tag{13}$$

5 Second approach: Differentiable optimization

We denote by Ω the closed and convex subset of $\mathbb{R}^6$ consisting of all the points $s \in \mathbb{R}^6$ satisfying the constraints (6)-(8), that is, defining $l_1 = l_3 = l_5 = x_{min}$, $u_1 = u_3 = u_5 = x_{max}$, $l_2 = l_4 = l_6 = y_{min}$, $u_2 = u_4 = u_6 = y_{max}$, the admissible set Ω is given by

$$\begin{aligned}\Omega = \{s = (s_1, s_2, \dots, s_6) \in \mathbb{R}^6 \ :\ &l_i \leq s_i \leq u_i,\ i = 1, 2, \dots, 6,\\ &s_3 - s_1 \geq h_1,\ s_2 - s_4 \geq h_2,\ s_1 - s_5 \geq d_1,\ s_6 - s_2 \geq d_2\}\end{aligned} \tag{14}$$

Then, our original problem $(\mathscr{P})$ can be rewritten as:

$$\min_{s\in\Omega} J(s) \tag{15}$$

In order to solve this problem, we also propose a Spectral Projected-Gradient algorithm (SPG) due to Birgin *et al.* [7] (providing in each iteration an admissible point in Ω) where global convergence is assured under reasonable hypotheses (see [7] for details). So, assuming the gradient ∇J to be available in each iteration, the SPG algorithm can be summarized into the following steps:
Step 0. (Initialization) Let $\bar{s} \in \Omega$, and let $\varepsilon > 0$ be a positive tolerance.
Step 1. (Search direction computation) Let $d = P_\Omega(\bar{s} - \eta \nabla J(\bar{s})) - \bar{s}$, where $\eta > 0$ is given by:

- First iteration: $\eta = 1$,
- Subsequent other iterations: Let $\bar{s}$ be the current point and $\widetilde{s}$ the previous point. Compute $x = \bar{s} - \widetilde{s}$ and $y = \nabla J(\bar{s}) - \nabla J(\widetilde{s})$. Then, if $x^T y > 0$, take $\eta = \frac{x^T x}{x^T y}$; elsewhere, take η as a fixed positive value.

Furthermore, $y = P_\Omega(z)$ represents the projection y of $z \in \mathbb{R}^6$ onto Ω that can be computed in an easy way (due to the special characteristics of our admissible set Ω) as the stationary point (that is, the global minimum) of the strictly convex quadratic function minimizing the distance from z to Ω:

$$\min_{y\in\Omega} \frac{1}{2}\|y-z\|_2^2 = \min_{y\in\Omega} \frac{1}{2} z^T z - z^T y + \frac{1}{2} y^T y \tag{16}$$

Step 2. (Termination) If $d = 0$ (in practice, $\|d\|_2 < \varepsilon$), then stop: $\bar{s}$ is a stationary point of J on Ω.
Step 3. (Stepsize) Compute a value $\alpha \in (0,1]$ such that $J(\bar{s}+\alpha d) \le J(\bar{s}) + \alpha\beta\nabla J(\bar{s})^T d$, with $\beta > 0$ (usually, $\beta \in [10^{-4}, 10^{-1}]$). So, in order to compute the stepsize α in an iterative way, choose $\gamma > 0$ (usually, $\gamma = 2$), take $\theta_1 = J(\bar{s})$ and $\theta_2 = \beta\nabla J(\bar{s})^T d$, and then, for $p = 0,1,2,\ldots$, define $\alpha = \frac{1}{\gamma^p}$, and stop when $J(\bar{s}+\alpha d) \le \theta_1 + \alpha\theta_2$.
Step 4. (Update) Define $\widetilde{s} = \bar{s} + \alpha d$, and go to Step 1 with $\bar{s} = \widetilde{s}$.

In this algorithm, the values of function $J(\bar{s})$ can be directly obtained from expression (5) in previous sections. An expression for the gradient could also be obtained by solving the adjoint state system (see [4]). However, this expression is too cumbersome for our problem and, in general, not useful in practice. We propose to approximate the gradient of J by a finite difference approach:

For a fixed $\bar{s} \in \Omega$, the gradient $\nabla J(\bar{s}) = \left(\frac{\partial J}{\partial s_1}(\bar{s}), \frac{\partial J}{\partial s_2}(\bar{s}), \ldots, \frac{\partial J}{\partial s_6}(\bar{s})\right)$ can be approximated, for $\delta > 0$ small enough, by:

$$\frac{\partial J}{\partial s_i}(\bar{s}) \approx \frac{J(\bar{s}+\delta e_i) - J(\bar{s})}{\delta}, \qquad i = 1,2,\ldots,6.$$

In this way, once the value of $J(\bar{s})$ is obtained from (5), in order to compute $\nabla J(\bar{s})$, only six function evaluations of J are required. Computation of the stepsize only needs one function evaluation of J, for each trial.

Finally, the quadratic problem (16) giving the projection $y = P_\Omega(z)$, can be uncoupled into two simpler quadratic problems. In fact,

$$\left.\begin{array}{ll} \displaystyle\min_{(y_1,y_2,\ldots,y_6)} & \displaystyle\sum_{i=1}^{6}\{\tfrac{1}{2}z_i^2 - z_i y_i + \tfrac{1}{2}y_i^2\} \\ \text{subject to} & l_i \le y_i \le u_i,\ i = 1,2,\ldots,6, \\ & y_3 - y_1 \ge h_1, \quad y_2 - y_4 \ge h_2, \\ & y_1 - y_5 \ge d_1, \quad y_6 - y_2 \ge d_2. \end{array}\right\} \Longleftrightarrow$$

$$\left\{\begin{array}{l} \begin{array}{ll} \displaystyle\min_{(y_1,y_3,y_5)} & \tfrac{1}{2}(z_1^2+z_3^2+z_5^2) - (z_1y_1+z_3y_3+z_5y_5) + \tfrac{1}{2}(y_1^2+y_3^2+y_5^2) \\ \text{subject to} & l_i \le y_i \le u_i,\ i = 1,3,5, \\ & y_1 - y_5 \ge d_1, \quad y_3 - y_1 \ge h_1, \end{array} \\ \text{and} \\ \begin{array}{ll} \displaystyle\min_{(y_2,y_6,y_4)} & \tfrac{1}{2}(z_2^2+z_6^2+z_4^2) - (z_2y_2+z_6y_6+z_4y_4) + \tfrac{1}{2}(y_2^2+y_6^2+y_4^2) \\ \text{subject to} & l_i \le y_i \le u_i,\ i = 2,6,4, \\ & y_2 - y_4 \ge h_2, \quad y_6 - y_2 \ge d_2. \end{array} \end{array}\right.$$

Since each one of the above quadratic problems has only three dimensions, they can easily be solved by using the Karush-Kuhn-Tucker (KKT) conditions. For instance, the first uncoupled problem corresponding to variables (y_1, y_3, y_5) (for the second problem involving variables (y_2, y_6, y_4), the argumentation is completely analogous), we must remark that the four bound constraints $l_1 \leq y_1 \leq u_1$, $y_3 \geq l_3$, $y_5 \leq u_5$ are redundant (as a straight consequence of the other non-bound constraints (7) and (8) and due to the fact that $l_1 = l_3 = l_5$, $u_1 = u_3 = u_5$). Thus, from the original eight constraints, only four linear constraints are left for the optimization problem. In this way, these two quadratic problems can be rewritten as:

$$\begin{aligned} \min_{(x_1,x_2,x_3)} \;& \alpha - (a_1x_1 + a_2x_2 + a_3x_3) + \tfrac{1}{2}\left(x_1^2 + x_2^2 + x_3^2\right) \\ \text{subject to } & x_2 \leq u, \\ & x_3 \geq l, \\ & x_1 - x_3 \geq d, \\ & x_2 - x_1 \geq h, \end{aligned}$$

where u, l, d, h, α and a_i, for $i = 1,2,3$, are real numbers. Let $w = (w_1, w_2, w_3, w_4)^T$ be the vector of the Lagrange multipliers associated to the four constraints. Furthermore, let

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} -u \\ l \\ d \\ h \end{bmatrix},$$

and write the linear constraints in the form

$$Ax - v = b, \;\; v \geq 0,$$

where $v = (v_1, v_2, v_3, v_4)^T$ is the vector of the slack variables. Since the objective function is strictly convex and quadratic in $\mathbb{R}^3$ and the linear constraints are consistent, the optimization problem has a unique (global) optimal solution that satisfies the KKT conditions:

$$\begin{gathered} v = Ax - b, \\ -a + x = A^T w, \\ v \geq 0, \; w \geq 0, \\ v^T w = 0, \end{gathered}$$

where $a = (a_1, a_2, a_3)^T$. Thus, the optimal solution is given by

$$x = a + A^T w$$

that is,

$$\begin{cases} x_1 = a_1 + w_3 - w_4, \\ x_2 = a_2 - w_1 + w_4, \\ x_3 = a_3 + w_2 - w_3. \end{cases}$$

To compute the Lagrange multipliers w_i, we eliminate the unrestricted variables x_i to get the following linear complementarity problem (LCP):

$$\begin{gathered} v = (-b + Aa) + AA^T w, \\ v \geq 0, \; w \geq 0, \\ v^T w = 0. \end{gathered}$$

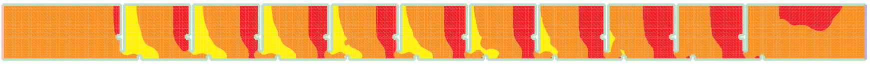

Fig. 4 Initial fishway and corresponding water height at final time $T = 300s$ for random point: $a = (0.6, 0.15)$, $b = (0.9, 0.07)$, $c = (0.5, 0.4)$. Height values: 0.26 (blue), 0.38 (green), 0.46 (yellow), 0.50 (orange), 0.54 (red).

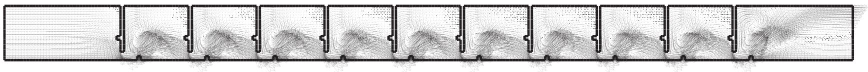

Fig. 5 Initial horizontal velocity field at final time $T = 300s$ for random point: $a = (0.6, 0.15)$, $b = (0.9, 0.07)$, $c = (0.5, 0.4)$.

Let

$$q = -b + Aa = (u - a_2, a_3 - l, a_1 - a_3 - d, -a_1 + a_2 - h)^T$$

and

$$M = AA^T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}.$$

Since M is a Z-matrix (all off-diagonal elements are non-positive), then this LCP(q,M) can be solved by the so-called Chandrasekaran's algorithm [21]:
Step 0. Let $w = 0$, $v = q$ and $I = \{i \ : \ v_i < 0\}$. If $I = \emptyset$, then $w = 0$ is a solution of the LCP.
Step 1. Consider the system of linear equations

$$M_{II} w_I = -q_I$$

If M_{II} is singular then the LCP is infeasible. Otherwise, let $\overline{w}_I$ be the unique solution of this linear system and set

$$\overline{w}_j = 0, \ \ \forall j \notin I.$$

Step 2. Compute

$$\begin{cases} \overline{v}_i = 0, & \text{if } \ i \in I, \\ \overline{v}_i = q_i + \sum_{j \in I} m_{ij} \overline{w}_j, & \text{if } \ i \notin I, \end{cases}$$

and let $J = \left\{ j \ : \ \overline{v}_j < 0 \right\}$.
Step 3. If $J = \emptyset$, then $\overline{w}$ is a solution of the LCP. Otherwise go to Step 1 with $I = I \cup J$.

It is also worthwhile remarking here that this algorithm is polynomially bounded as the number of iterations is smaller than or equal to 4. Furthermore the case of infeasible LCP in Step 1 cannot occur, as the theory shows that this LCP has a unique solution.

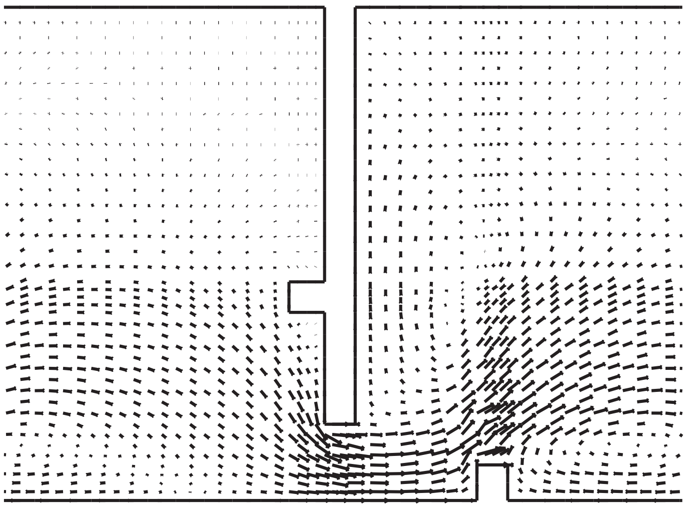

Fig. 6 Initial horizontal velocity field in the central pool at final time $T = 300\,s$.

6 Numerical results

In this section we present several numerical results obtained for a standard situation. We have considered the fishway under study, whose scheme is shown in Fig. 2. Both initial and boundary conditions were held constant, particularly, $H_0 = 0.5\,m$, $\mathbf{Q}_0 = (0,0)\,m^2s^{-1}$, $q_1 = -\frac{0.065}{0.97}\,m^2s^{-1}$, $H_2 = 0.5\,m$. The time interval for the simulation was $T = 300\,s$. Moreover, for the sake of simplicity, for the second member $\mathbf{f}$ we have only considered the bottom friction stress for a Chezy coefficient of $57.36\,m^{0.5}s^{-1}$. For the objective function we chose a target velocity value $c = 0.8\,ms^{-1}$, and a weight parameter $\xi = 0$. Finally, for the time discretization we chose $N = 3000$ (that is, a time step of $\Delta t = 0.1\,s$) and, for the different space discretizations, we tried several regular triangulations of about 9500 elements.

Although we have developed many numerical experiences, we present here only a couple of examples for this realistic problem. However, we can point out the following essential remarks. The initial starting point for the optimization algorithm shows a great influence in the computed optimal solution. Starting from different initial points we arrive to different optimal configurations (local minima), but all of them show very similarly acceptable levels of fulfilment. As far as the sensitivity of the solution to each parameter is concerned, all the six parameters seem to be equally important, since in our experiments we could not rapidly fix any parameter to speed up optimization. Moreover, we observed how the optimal configurations show a velocity profile very close to the target velocity: a velocity parallel to the side wall in the slot zone (except around the ends of baffles that should be, obviously, avoided), and almost null in the remaining of the fishway. We can also note that this is not true for the non-optimal configurations. Finally, we can also observe how the recirculation areas, clearly noticed in the initial configuration, completely disappear in the optimal configuration, providing a significantly improved design.

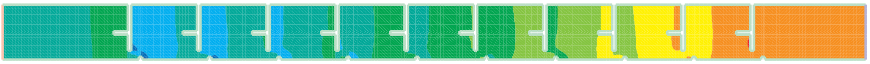

Fig. 7 NM algorithm: Optimal fishway and corresponding water height at final time $T = 300\,s$ for optimal point: $a_{NM} = (0.7248, 0.1573)$, $b_{NM} = (0.9169, 0.0494)$, $c_{NM} = (0.4450, 0.4727)$. Height values: 0.26 (blue), 0.38 (green), 0.46 (yellow), 0.50 (orange), 0.54 (red).

Experiment 1: NM algorithm

In this first example, we used a penalty parameter $\mu = 10^5$. Applying the NM algorithm we passed, after 167 function evaluations, from initial cost J (as shown in Table 1) for point ♯1 of a random simplex, to the minimum cost $J = 240.4255$, corresponding to the optimal design variables $a_{NM} = (0.7248, 0.1573)$, $b_{NM} = (0.9169, 0.0494)$, $c_{NM} = (0.4450, 0.4727)$. The complete process took a total of about 99 hours of CPU time in a laptop with two Intel Pentium 4 microprocessors. Figs. 4 and 5 show, respectively, the water height at the final time of the simulation (representing the stationary situation) and the water velocity corresponding to the fishway given by the initial random configuration for point ♯1 (see Table 1). Figs. 7 and 8 show the water height and the water velocity in the whole fishway at the final time of the simulation corresponding to the optimal configuration given by a_{NM}, b_{NM} and c_{NM}. In both cases we observed that the flow structure is very similar in each pool of each fishway and that, in the optimal case, a clearly defined streamline appears passing through all of the vertical slots.

Two close-ups of the central pool are shown, respectively, in Figs. 6 and 9. In the case of the initial shape (Fig. 6) we can identify the standard flow patterns presented, for instance, in Rajaratnam *et al.* [27]: a direct flow region where the flow circulates in a curved trajectory at high velocity from one slot to the next downstream, and two recirculation regions - the larger one located between the long baffles and the smaller one located between the short baffles - flowing in opposite directions. In the case of the optimal shape (Fig. 9) the direct flow velocity is very close to the target horizontal velocity **v**, the smaller recirculation region is completely removed, and the larger one is highly reduced. Finally, comparing Figs. 4 and 7, we can also see how, in the initial case (Fig. 4) the eddy areas create great differences in the height of water inside each pool, but in the optimal case (Fig. 7) these variations are much smoother.

Table 1 Initial random 7-simplex for NM algorithm

point ♯	s_1	s_2	s_3	s_4	s_5	s_6	$J(s)$
1	0.6	0.15	0.9	0.07	0.5	0.4	551.0569
2	0.4	0.13	0.9	0.03	0.31	0.3	450.5361
3	0.5	0.16	0.7	0.1	0.35	0.4	649.5730
4	0.6	0.15	0.85	0.08	0.4	0.25	533.9816
5	0.7	0.15	0.9	0.05	0.4	0.45	275.9281
6	0.5	0.24	0.55	0.04	0.4	0.3	$> 10^5$
7	0.45	0.18	0.6	0.12	0.35	0.3	1025.8988

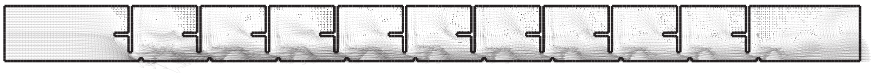

Fig. 8 NM algorithm: Horizontal velocity field at final time $T = 300\,s$ for optimal point: $a_{NM} = (0.7248, 0.1573)$, $b_{NM} = (0.9169, 0.0494)$, $c_{NM} = (0.4450, 0.4727)$.

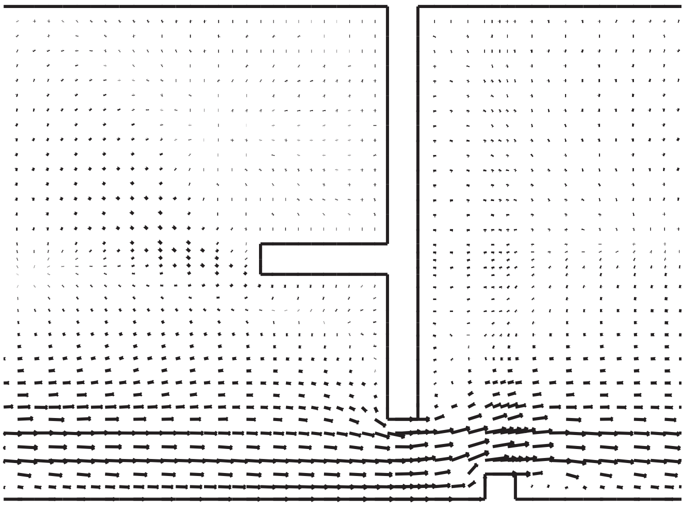

Fig. 9 NM algorithm: Horizontal velocity field in the central pool at final time $T = 300\,s$.

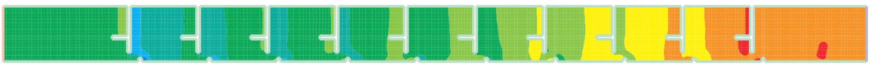

Fig. 10 SPG algorithm: Optimal fishway and corresponding water height at final time $T = 300\,s$ for optimal point: $a_{SPG} = (0.7032, 0.1593)$, $b_{SPG} = (0.9002, 0.0633)$, $c_{SPG} = (0.4076, 0.4238)$. Height values: 0.26 (blue), 0.38 (green), 0.46 (yellow), 0.50 (orange), 0.54 (red).

Experiment 2: SPG algorithm

In this second example, we chose a stepsize $\delta = 10^{-3}$, a spectral parameter $\eta = 10^{15}$ and a tolerance $\varepsilon = 10^{-3}$. Applying the SPG algorithm, we passed, after only 5 iterations, from initial cost $J = 275.9281$, corresponding to the point $\sharp 5$ $a = (0.7, 0.15)$, $b = (0.9, 0.05)$, $c = (0.4, 0.45)$ (the "best" point of above initial random simplex), to the minimum cost $J = 242.6674$, corresponding to the optimal design variables $a_{SPG} = (0.7032, 0.1593)$, $b_{SPG} = (0.9002, 0.0633)$, $c_{SPG} = (0.4076, 0.4238)$. In this case, the total CPU time was reduced to 27.4 hours (nearly a 72% reduction with respect to experiment 1). Figs. 10-12 show the water height and the water velocity in the whole fishway at the final time of the simulation corresponding to the optimal configuration given by a_{SPG}, b_{SPG} and c_{SPG}. The same remarks on experiment 1 (showing very similar results) apply to this example.

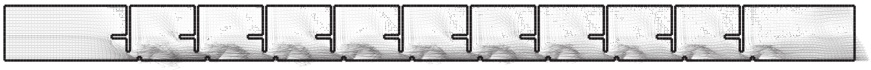

Fig. 11 SPG algorithm: Horizontal velocity field at final time $T = 300s$ for optimal point: $a_{SPG} = (0.7032, 0.1593)$, $b_{SPG} = (0.9002, 0.0633)$, $c_{SPG} = (0.4076, 0.4238)$.

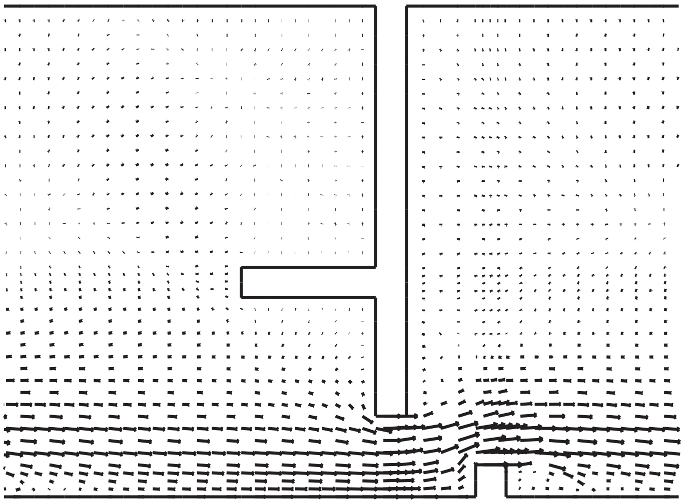

Fig. 12 SPG algorithm: Horizontal velocity field in the central pool at final time $T = 300s$.

7 Conclusions

Mathematical modelling has been used to simulate height and velocity of the water in a standard vertical slot fishway. Moreover, optimization and optimal control techniques have been employed to control the water velocity. Particularly, we have obtained a more suitable shape design for fishways, which is optimal in terms of swimming capabilities and rest necessities for fish when passing through the fishway.

After obtaining a well-posed formulation for the environmental problem, we have proposed two different approaches in order to achieve the optimal shape. In the first approach, we used a direct search method (NM algorithm). In the second approach, we used the SPG method, where the derivatives were computed *via* a finite difference approximation. Analyzing the numerical experiences developed by the authors, we conclude that both algorithms are effective, robust and reliable. The cost value achieved in the example is slightly better for the optimal shape obtained by the NM algorithm, but, as a counterpart, the computational effort is clearly lower for the SPG method.

It is worthwhile remarking here that the optimization problem is non-convex, so there exist several local minima with different optimal values. Our experiences show that both algorithms determine different local minima. Nevertheless, both local minima produce design solutions that are interesting and satisfactory, from a practical viewpoint, for the model under study.

Finally, it is important to emphasize that the NM algorithm presents computational difficulties when applied to a more complex model with a larger number of design variables. It is this inability of the direct method to deal with these problems that led to our proposal of using the projected-gradient algorithm. In fact, if the constraints of the model are simple, as in our problem, special-purpose techniques may be designed for the computation of the projection and the SPG method will be able to solve efficiently the optimization problem related to the model.

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The financial support provided by Project MTM2009-07749 of Ministerio de Ciencia e Innovación (Spain), and Project INCITE09PXIG291083PR of Xunta de Galicia is gratefully acknowledged.

References

1. Alvarez-Vázquez LJ, Martínez A, Muñoz-Sola R, Rodríguez C, Vázquez-Méndez ME, The water conveyance problem: Optimal purification of polluted waters, Math Models Meth Appl Sci 15:1393-1416 (2005)
2. Alvarez-Vázquez LJ, Martínez A, Rodríguez C, Vázquez-Méndez ME, Numerical optimization for the location of wastewater outfalls, Comput Optim Appl 22:399-417 (2002)
3. Alvarez-Vázquez LJ, Martínez A, Rodríguez C, Vázquez-Méndez ME, Vilar MA, Optimal shape design for fishways in rivers, Math Comput Simul 76:218-222 (2007)
4. Alvarez-Vázquez LJ, Martínez A, Vázquez-Méndez ME, Vilar MA, An optimal shape problem related to the realistic design of river fishways, Ecological Eng 32:293-300 (2008)
5. Alvarez-Vázquez LJ, Martínez A, Vázquez-Méndez ME, Vilar MA, Vertical slot fishways: modeling and optimal management, J Comput Appl Math 218:395-403 (2008)
6. Bermúdez A, Rodríguez C, Vilar MA, Solving shallow water equations by a mixed implicit finite element method, IMA J Numer Anal 11:79-97 (1991)
7. Birgin G, Martínez JM, Raydan M, Nonmonotone spectral projected gradient methods on convex sets, SIAM J Optim 10:1196-1211 (2000)
8. Blake RW, Fish locomotion. Cambridge University Press, London (1983)
9. Chorda J, Maubourguet MM, Roux H, Larinier M, Tarrade L, David L, Two-dimensional free surface flow numerical model for vertical slot fishways, J Hydraul Res 48:141-151 (2010)
10. Clay CH, Design of fishways and other fish facilities. Lewis Publishers, CRC Press, Boca Raton (1995)
11. Ead SA, Rajaratnam N, Katopodis C, Generalized study of hydraulics of culvert fishways, J Hydraul Eng 128:1018-1022 (2002)
12. Ead SA, Katopodis C, Sikora GJ, Rajaratnam N, Flow regimes and structure in pool and weir fishways, J Environ Eng Sci 3:379-390 (2004)
13. Karisch SE, Power M, A simulation study of fishway design: an example of simulation in environmental problem solving, J Environ Management 41:67-77 (1994)
14. Katopodis C, Rajaratnam N, Wu S, Towell D, Denil fishways of varying geometry, J Hydraul Eng 123:624-631 (1997)
15. Kelley CT, Detection and remediation of stagnation in the Nelder-Mead algorithm using a sufficient decrease condition, SIAM J Optim 10:43-55 (1999)
16. Kim JH, Hydraulic characteristics by weir type in a pool-weir fishway, Ecological Eng 16:425-433 (2001)
17. Lagarias JC, Reeds JA, Wright MH, Wright PE, Convergence properties of the Nelder-Mead simplex algorithm in low dimensions, SIAM J Optim 9:112-147 (1998)
18. Liu M, Rajaratnam N, Zhu DZ, Mean flow and turbulence structure in vertical slot fishways, J Hydraul Eng 132:765-777 (2006)

19. Mallen-Cooper M, Stuart IG, Optimising Denil fishways for passage of small and large fishes, Fisheries Management Ecol 14:61-71 (2007)
20. Martínez A, Rodríguez C, Vázquez-Méndez ME, Theoretical and numerical analysis of an optimal control problem related to wastewater treatment, SIAM J Control Optim 38:1534-1553 (2000)
21. Murty K, Linear Complementarity, Linear and Nonlinear Programming. Heldermann Verlag, Berlin (1988)
22. Nelder JA, Mead R, A simplex method for function minimization, Comput J 7:308-313 (1965)
23. Odeh M, Discharge rating equation and hydraulic characteristics of standard Denil fishways, J Hydraul Eng 129:341-348 (2003)
24. Puertas J, Pena L, Teijeiro T, Experimental approach to the hydraulics of vertical slot fishways, J Hydraul Eng 130:10-23 (2004)
25. Rajaratnam N, Katopodis C, Hydraulics of Denil fishways, J Hydraul Eng 110:1219-1233 (1984)
26. Rajaratnam N, Katopodis C, Hydraulics of steeppass fishways, Can J Civil Eng 18:1024-1032 (1991)
27. Rajaratnam N, Van de Vinne G, Katopodis C, Hydraulics of vertical slot fishways, J Hydraul Eng 112:909-917 (1986)
28. Rajaratnam N, Katopodis C, Mainali A, Plunging and streaming flows in pool and weir fishways, J Hydraul Eng 114:939-944 (1988)
29. Rajaratnam N, Katopodis C, Solanski S, New designs for vertical slot fishways, J Hydraul Eng 119:402-414 (1992)
30. Rajaratnam N, Katopodis C, Wu S, Sabur MA, Hydraulics of resting pools for Denil fishways, J Hydraul Eng 123:632-638 (1997)
31. Richmond MC, Deng Z, Guensch GR, Tritico H, Pearson WH, Mean flow and turbulence characteristics of a full-scale spiral corrugated culvert with implications for fish passage, Ecological Eng 30:333-340 (2007)
32. Weber NS, Joy DM, Use of a scale model to design fishway resting pool improvements, Can Water Res J 27:401-426 (2002)
33. Wu S, Rajaratnam N, Katopodis C, Structure of flow in vertical slot fishway, J Hydraul Eng 125:351-360 (1999)
34. Yasuda Y, Ohtsu I, Takahashi M, New portable fishway design for existing trapezoidal weirs, J Environ Eng Sci 3:391-401 (2004)